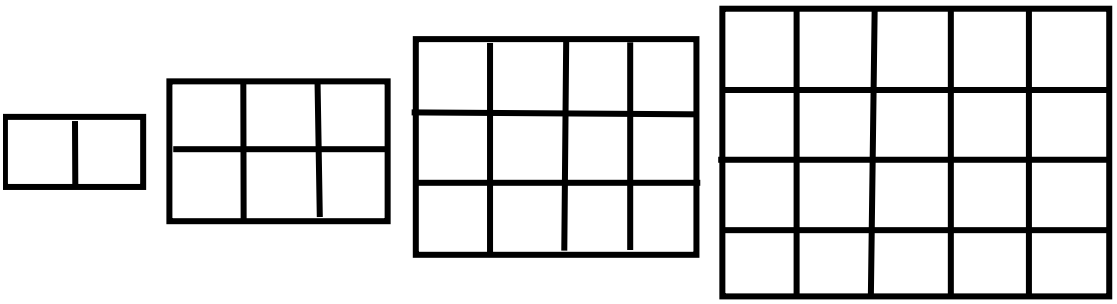
IMLEM Meet #3
January, 2014

Intermediate Mathematics League of Eastern Massachusetts

50th anniversary edition

Category 1
Mystery
Meet #3 - January, 2014 *50th anniversary edition*

1) The first rectangle below has two unit squares. The second has six, the third has 12, the fourth has 20, and so on. If the pattern continues, then how many unit squares are in the 20th rectangle ?



2) If X Ω Y means "multiply Y by 3 and then add X" and X ® Y means "subtract X from 25 and then subtract that answer from Y," then find the value of (3 Ω 5) ® 10.

3) Definition of midpoint: Point D is the midpoint of segment EF if D lies on EF and ED = DF.
Points A, B, and C lie on segment LP in such a way that A is the midpoint of LP, B is the midpoint of LA, and C is the midpoint of AP. If AC = 5, then what is the length of BP?

ANSWERS

1) _____

2) _____

3) _____

Category 1 - Solutions
Mystery
Meet #3 - January, 2014

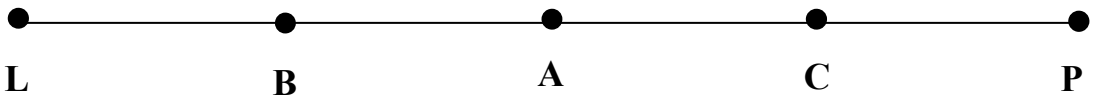
ANSWERS
1) 420
2) 3
3) 15

1) The first rectangle is a 1x2, the second is a 2x3, the third a 3x4, the fourth a 4x5, and . . . the Nth is N by (N+1). When N is 20, there are 20x21, or 420, unit squares in the rectangle.

2) $(3 \Omega 5)$
$= 3x5 + 3$
$= 15 + 3$
$= 18$

$(3 \Omega 5) \circledR 10$
$= (18) \circledR 10$
$= 10 - (25 - 18)$
$= 10 - 7$
$= 3$

3)

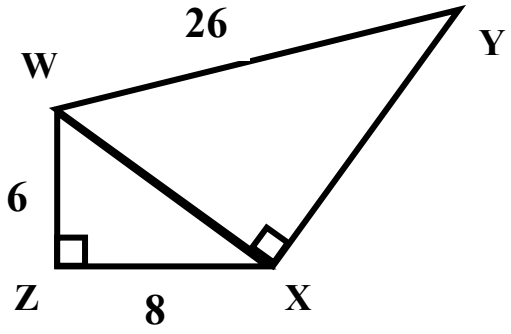


Since AC = 5, and BP consists of three such segments in succession, then BP = 15.

Category 2 *50th anniversary edition*

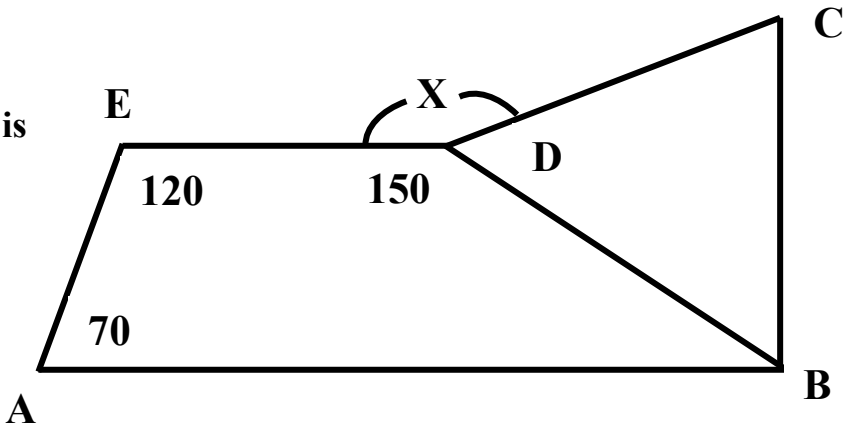
Geometry

Meet #3 - January, 2014

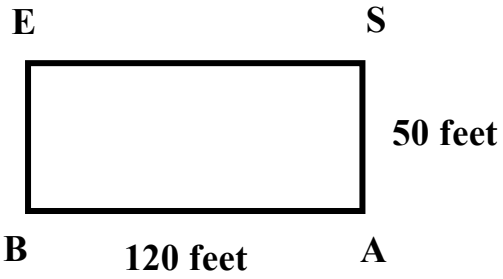


1) How many cm long is segment XY ? All measurements are in centimeters (cm).

2) Angle ABC is a right angle. Triangle BCD is an isosceles triangle such that DB = BC. Find the value of X if it is the measure in degrees of angle EDC and X < 180.



3) Moe and Larry race from point B to point S at a rectangular field. Moe runs from B to A to S at an average rate of 5 feet every second. Larry runs diagonally across the field from B to S at an average rate of 10 feet every 3 seconds. If they both leave point B at the same time, then who wins the race? Also, by how many seconds does the winner finish ahead of the runner-up? (You must answer both questions correctly to receive credit.)



ANSWERS

1) _____ cm

2) _____

3) _____ winner

_____ seconds

Solutions to Category 2
Geometry
Meet #3 - January, 2014

Answers
1) 24
2) 155
3) Moe 5

1) Use the Pythagorean Theorem twice - first to find the length of WX and then XY.

$$6^2 + 8^2 = (WX)^2$$
$$36 + 64 = (WX)^2$$
$$100 = (WX)^2$$
$$10 = WX$$

Use this result to find XY:

$$10^2 + (XY)^2 = 26^2$$
$$100 + (XY)^2 = 676$$
$$(XY)^2 = 576$$
$$XY = 24$$

2) The measure of angle DBA is 20 degrees, because the sum of the angles of a quadrilateral is 360 degrees.
The measure of angle DBC is 70 degrees, because angle ABC is a right angle (90 degrees).
Since two sides (DB and BC) of triangle DBC are congruent, the angles opposite those sides are congruent. The vertex angle, DBC, measures 70 degrees, so the base angles are 55 degrees each, including angle BDC. angle X + 150 + 55 = 360, so X = 155.

3) Both answers must be answered correctly in order for students to receive credit.
Moe: runs 120 + 50, or 170 feet. At a rate of 5 feet per second, it takes him 170 / 5, or 34 seconds to reach point S.
Larry: Use the Pythagorean Theorem to find that he has run 130 feet. At a rate of 10 feet every 3 seconds, it takes him (130 / 10) x 3, or 39 seconds to reach point S.
Moe, therefore, reaches point S ahead of Larry by 39 - 34, or by 5 seconds, so Moe wins the race.

Category 3
Number Theory
Meet #3 - January, 2014 *50th anniversary edition*

1) The numeral 1101001 is written in base 2. Write it in base 10.

2) The average strep bacterium has a diameter of about 90×10^{-6} of a meter while the average flu virus has a diameter of about 0.2×10^{-4} of a meter. How many times greater is the diameter of an average strep bacterium than an average flu virus? Express your answer in scientific notation.

3) Evaluate. Write your answer in scientific notation.

$$\frac{72 \times 10^{7}}{1.2 \times 10^{0}} \times \frac{240 \times 10^{-6}}{0.09 \times 10^{5}} \div \frac{0.0008 \times 10^{-2}}{400 \times 10^{12}}$$

Answers

1) _____ base 10

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #3 - January, 2014

Answers

1) 105

2) 4.5×10^0

3) 8×10^{20}

1) from right to left:

$(1x2^0)+(0x2^1)+(0x2^2)+(1x2^3)+(0x2^4)+(1x2^5)+(1x2^6)$

$= 1+0+0+8+0+32+64$

$= 105$

2) Divide the larger by the smaller: $\dfrac{90 \times 10^{-6}}{0.2 \times 10^{-4}}$

$= \dfrac{9.0 \times 10^{-5}}{2 \times 10^{-5}}$

$= 4.5 \times 10^0$

3) $\dfrac{72 \times 10^7}{1.2 \times 10^0} \times \dfrac{240 \times 10^{-6}}{0.09 \times 10^5} \div \dfrac{0.0008 \times 10^{-2}}{400 \times 10^{12}}$

$= \dfrac{7.2 \times 10^8}{1.2 \times 10^0} \times \dfrac{2.4 \times 10^{-4}}{9 \times 10^3} \div \dfrac{8 \times 10^{-6}}{4 \times 10^{14}}$

$= \dfrac{7.2 \times 10^8}{1.2 \times 10^0} \times \dfrac{2.4 \times 10^{-4}}{9 \times 10^3} \times \dfrac{4 \times 10^{14}}{8 \times 10^{-6}}$

$= \dfrac{7.2 \times 2.4 \times 4 \times 10^8 \times 10^{-4} \times 10^{14}}{1.2 \times 9 \times 8 \times 10^0 \times 10^3 \times 10^{-6}}$

$= 0.8 \times 10^{21}$

$= 8 \times 10^{20}$

Category 4
Arithmetic

Meet #3 - January, 2014 *50th anniversary edition*

1) Evaluate: $2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2}$ **Express your answer as a decimal.**

2) If $5^4 + 5^3 - 5^2 + 5^1 - 5^0 = 3^N$, **then find the value of N.**

3) Evaluate: $\sqrt[3]{\sqrt{\sqrt{12^2} \times \sqrt[4]{81}} \times \sqrt[3]{64} \times \sqrt[5]{243} \times \sqrt{9}}$

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #3 - January, 2014

Answers
1) 7.75
2) 6
3) 6

1) $2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2}$

$= 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$

$= 7.75$

2) $5^4 + 5^3 - 5^2 + 5^1 - 5^0 = 3^N$

$625 + 125 - 25 + 5 - 1 = 3^N$

$729 = 3^N$

$N = 6$

3) $\sqrt[3]{\sqrt{\sqrt{12^2} \times \sqrt[4]{81} \times \sqrt[3]{64} \times \sqrt[5]{243} \times \sqrt{9}}}$

$= \sqrt[3]{\sqrt{12 \times 3} \times 4 \times 3 \times 3}$

$= \sqrt[3]{\sqrt{36} \times 36}$

$= \sqrt[3]{6 \times 36}$

$= \sqrt[3]{216}$

$= 6$

Category 5
Algebra
Meet #3 - January, 2014 *50th anniversary edition*

1) If $|C| = 5$, then what is the sum of all values of C that make this sentence true?

2) The graph below represents the set of all values of X that make $|X - N| \leq 8$ true. What is the value of N ?

3) If $7 + 3(2Y - 5) < 52$ and $7 - 3(2Y - 5) < -8$

then the set of all possible values of Y is represented in the graph below:

What is the value of A + B ?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #3 - January, 2014

Answers
1) 0
2) 20
3) 15

1) The two solutions are 5 and -5.
Their sum is zero.

2) The absolute value sentence can be translated as, "The distance between a number, N, and all solutions is at most 8 units." N can be found by locating the midpoint of 12 and 28, which is 20.

3)
$$7 + 3(2Y - 5) < 52$$
$$7 + 6Y - 15 < 52$$
$$6Y - 8 < 52$$
$$6Y < 60$$
$$Y < 10$$

also,
$$7 - 3(2Y - 5) < -8$$
$$7 - 6Y + 15 < -8$$
$$22 - 6Y < -8$$
$$-6Y < -30$$
$$Y > 5$$

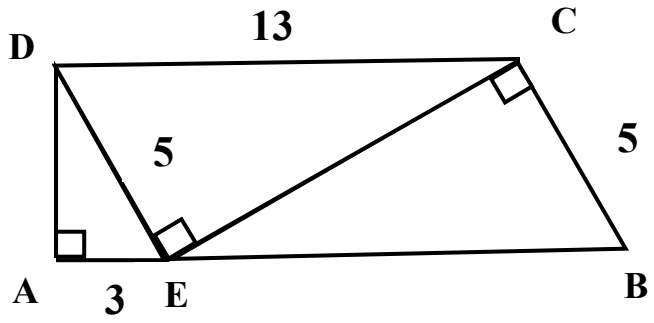
Therefore, A = 5 and B = 10 and their sum, A + B, is 5 + 10 = 15.

Category 6 *50th anniversary edition*

Team Round

Meet #3 - January, 2014

1) How many square cm are in the area of pentagon AEBCD? (AEB is *almost* a straight angle.) All measurements are in cm.



2) Write the base 4 numeral 2013 as a base 8 numeral.

3) Three lines intersect in the interior (inside) of a circle. What is the largest number of separate regions that can be made inside the circle with these lines?

4) If $2^X = 7$, then what is the value of 2^{X+4} ? (note: from yr 1963)

5) If $\frac{1}{3}|5X - 4| = 7$, then what is the product of the two possible values of X that makes the sentence true?

6) Using the answers from questions #1-5, evaluate the following expression:

$$\sqrt[5]{\left(\sqrt[3]{D-E-4}\right)^4 + 4B - AC + 33}$$

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

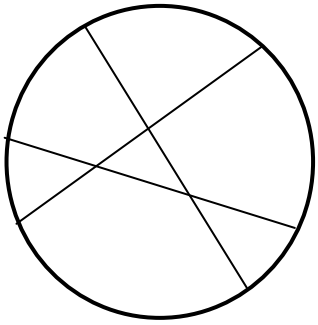
Solutions to Category 6
Team Round
Meet #2 - November, 2013

ANSWERS	
1)	66 = A
2)	207 = B
3)	7 = C
4)	112 = D
5)	- 17 = E
6)	4

1) Using the Pythagorean Theorem, EC = 12 and AD = 4. So, the area of pentagon AEBCD = the sum of the areas of the three triangles = 6 + 30 + 30 = **66**

2) Converting 2013 (base 4) to base 10:
3(1) + 1(4) + 0(16) + 2(64) = 3 + 4 + 0 + 128 = 135 (base 10).
Converting 135 (base 10) to base 8:
= 7(1) + 0(8) + 2(64)
= **207** (base 8).

3) The idea is to first draw two lines that intersect, then draw one line through the other two that does not pass through their point of intersection, as shown here. There are <u>seven</u> regions.



4) $2^X = 7$, so $2^{X+4} = 2^X(2^4) = (7)(16) = 112.$

5) $\frac{1}{3}|5X - 4| = 7$, so, either $|5X - 4| = 21$ or $|5X - 4| = -21$

$5X = 25$ or $5X = -17$, so $X = 5$ or $X = -17 / 5$

and $(5)(-17 / 5) = -17$

6) $\sqrt[5]{\left(\sqrt[3]{D - E - 4}\right)^4 + 4B - AC + 33} = \sqrt[5]{\left(\sqrt[3]{112 - (-17) - 4}\right)^4 + 4x207 - 66x7 + 33}$

$= \sqrt[5]{\left(\sqrt[3]{125}\right)^4 + 828 - 462 + 33} = \sqrt[5]{(5)^4 + 399} = \sqrt[5]{625 + 399} = \sqrt[5]{1024} = 4$