# Intermediate Mathematics League of Eastern Massachusetts

www.imlem.org

Meet #1
October 2003

# Category 1
# Mystery
# Meet #1, October 2003

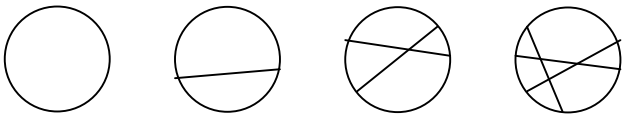
**1.** The first few rows of Pascal's Triangle are shown at right. If we call the top row the zeroth row and the left-most number in each row the zeroth number, then we would say that the value of the second number in the fifth row is 10, as circled. What is the value of the third number in the eighth row of Pascal's Triangle?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 (10) 10 5 1
1 6 15 20 15 6 1

**2.** The Big Boss at Sum Product & Company decided to round everyone's salary to the nearest thousand dollars. The salaries before the rounding are listed below. By how many dollars did the Big Boss reduce the total cost of salaries?

| | |
|---|---|
| Big Boss: | $72,508 |
| Right hand man: | $54,492 |
| Left hand woman: | $68,340 |
| Product designer: | $46,700 |
| Sum calculator: | $42,058 |

**3.** A circle that has not been cut is one piece. A circle with one straight cut is two pieces. A circle with two straight cuts is four pieces. A circle with three straight cuts can make seven pieces if the three lines do not cross at a single point. What is the largest number of pieces that can be cut from a circle with six straight cuts?



Answers

1. _____
2. _____
3. _____

# Solutions to Category 1
# Mystery
# Meet #1, October 2003

Answers

**1.** 56

**2.** 98

**3.** 22

**1.** The value of the third number in the eighth row of Pascal's Triangle is **56**, as shown below.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 (56) 70 56 28 8 1

**2.** The changes in salary are listed below:

| | | | | |
|---|---|---|---|---|
| Big Boss: | $72,508 | rounds to | $73,000 | a change of +$492 |
| Right hand man: | $54,492 | rounds to | $54,000 | a change of –$492 |
| Left hand woman: | $68,340 | rounds to | $68,000 | a change of –$340 |
| Product designer: | $46,700 | rounds to | $47,000 | a change of +$300 |
| Sum calculator: | $42,058 | rounds to | $42,000 | a change of –$058 |

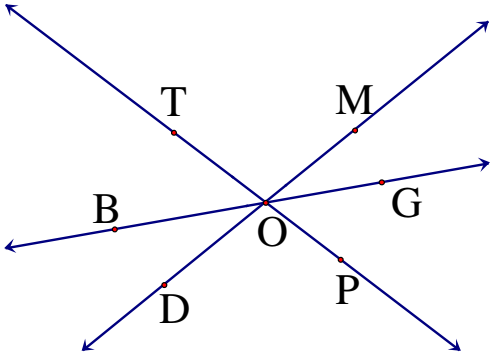
The total reduction in the cost of salaries is **$98**.

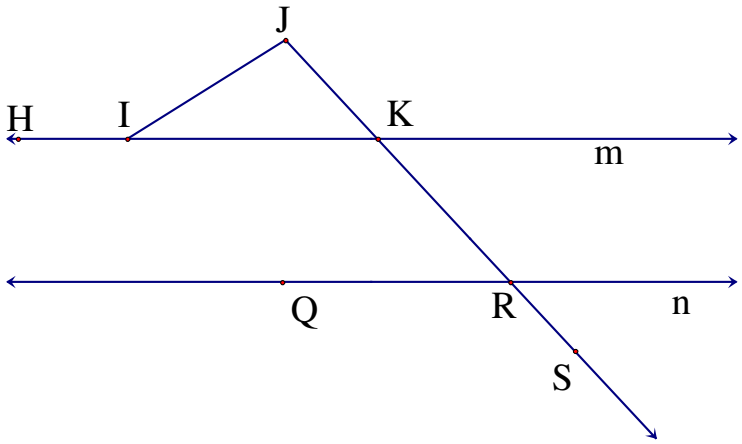
**3.** The table at right summarizes the number pattern. Though it is hard to draw, a circle can be cut into **22** pieces with just 6 cuts.

| Number of cuts | Number of pieces | First Difference | Second Difference |
|---|---|---|---|
| 0 | 1 | | |
| 1 | 2 | 1 | |
| 2 | 4 | 2 | 1 |
| 3 | 7 | 3 | 1 |
| 4 | 11 | 4 | 1 |
| 5 | 16 | 5 | 1 |
| 6 | **22** | 6 | 1 |

Category 2
Geometry
Meet #1, October 2003

**1.** Lines $\overleftrightarrow{TP}$, $\overleftrightarrow{BG}$, and $\overleftrightarrow{DM}$ intersect at point $O$. $m\angle BOT = 47$ degrees and $m\angle MOG = 29$ degrees. How many degrees are in the measure of angle $DOP$?




**2.** Lines $m$ and $n$ are parallel. $m\angle HIJ = 148$ degrees and $m\angle QRS = 133$ degrees. How many degrees are in the measure of angle $IJK$ if it is less than 180 degrees?

**3.** The sum of the supplement of angle $A$ and the complement of angle $A$ measures sixteen degrees more than a straight angle. How many degrees are in the measure of angle $A$?

Answers

1. _____
2. _____
3. _____

# Solutions to Category 2
# Geometry
# Meet #1, October 2003

Answers

**1.** 104

**2.** 101

**3.** 37

**1.** The measures of angles *BOT, TOM,* and *MOG* must add up to 180 degrees since *O* is a point on line *BG*. Thus the measure of angle *TOM* must be 180 – 47 – 29 = 104 degrees. Angles *TOM* and *DOP* are verticle angles and therefore have the same measure. The measure of angle *DOP* is **104** degrees.

**2.** Angle *HIJ* measures 148 degrees, so angle *JIK* must measure 180 – 148 = 32 degrees. Angle *QRS* and angle *IKR* are corresponding angles, so they have the same measure. This means angle *JKI* must measure 180 – 133 = 47. The total angle sum of triangle *IJK* has to be 180 degrees, so angle *IJK* must measure 180 – 32 – 47 = **101** degrees.

**3.** The supplement of angle *A* measures 180 – *A*. The complement of angle *A* measures 90 – *A*. Their sum is $(180-A)+(90-A)=270-2A$. If this amount is sixteen degrees more than a straight angle, then we can write the equation $270-2A=180+16$ and solve for *A*.

$$\begin{aligned} 270-2A &= 180+16 \\ 270-2A &= 196 \\ 270 &= 196+2A \\ 270-196 &= 2A \\ 74 &= 2A \\ A &= \mathbf{37} \end{aligned}$$

Category 3
Number Theory
Meet #1, October 2003

**1.** $9900 = 2^a \bullet 3^b \bullet 5^c \bullet 7^d \bullet 11^e$
Find the value of $a + b + c + d + e$.

**2.** Find the sum of all possible values of the digit *N* such that the 5-digit number 318*N*4 is divisible by 12.

**3.** Melanie's locker number is the product of the least pair of consecutive primes that have a difference of 6. What is Melanie's locker number?

Answers
1. _____
2. _____
3. _____

# Solutions to Category 3
# Number Theory
# Meet #1, October 2003

Answers

**1.** 7

**2.** 10

**3.** 667

**1.** $9900 = 2^2 \bullet 3^2 \bullet 5^2 \bullet 7^0 \bullet 11^1$,
so $a + b + c + d + e = 2 + 2 + 2 + 0 + 1 =$ **7**.

**2.** A number that is divisible by 12 must be divisible by both 3 and 4. For the 5-digit number 318*N*4 to be divisble by 3, the sum of its digits must be a multiple of 3. $3 + 1 + 8 + 4 = 16$, so *N* would have to be 2 or 5 or 8 to make a multiple of 3. This gives us 31824, 31854, and 31884 to consider. To determine which among these is divisible by 4, we need only check the last two digits of each number, since any multiple of 100 is divisble by 4. 24 and 84 are multiples of 4, but 54 is not. Thus only 31824 and 31884 are divisible by 12 and the sum of all possible values of *N* is 2 + 8 = **10**.

**3.** The least pair of consecutive primes with a difference of 6 is the pair 23 and 29. Their product is 667, so Melanie's locker number must be **667**

Category 4
Arithmetic
Meet #1, October 2003

**1.** Evaluate the following expression:

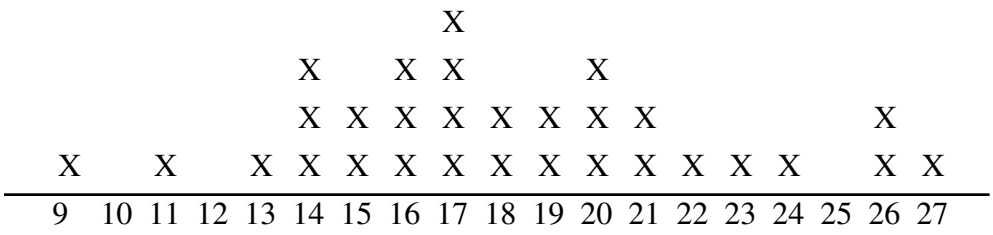
$$1\times\left(5^2-2^4\right)^2-2\times\left(5^2-2^4\right)^2+3\times\left(5^2-2^4\right)^2$$

**2.** The prize money at the raffle was given out as follows:

20 people received $5
10 people received $10
5 people received $20
2 people received $50
1 person received $100
1 person received $500
1 person received $1000

How much greater was the mean (average) of the prize money than the median?

**3.** The upper quartile is the median of the upper half of a set of data and the lower quartile is the median of the lower half. The interquartile range is the difference between the upper quartile and the lower quartile. Find the interquartile range of the data shown in the line plot below.




Answers
1. _____
2. _____
3. _____

# Solutions to Category 4

## Arithmetic
## Meet #1, October 2003

Answers

**1.** 162

**2.** $42.50

**3.** 6

**1.** It is helpful to notice that the same expression, $\left(5^2-2^4\right)^2$, appears in three places in the larger expression. Evaluating this by itself, according to the order of operations, we get:

$$\left(5^2-2^4\right)^2=(25-16)^2=9^2=81$$

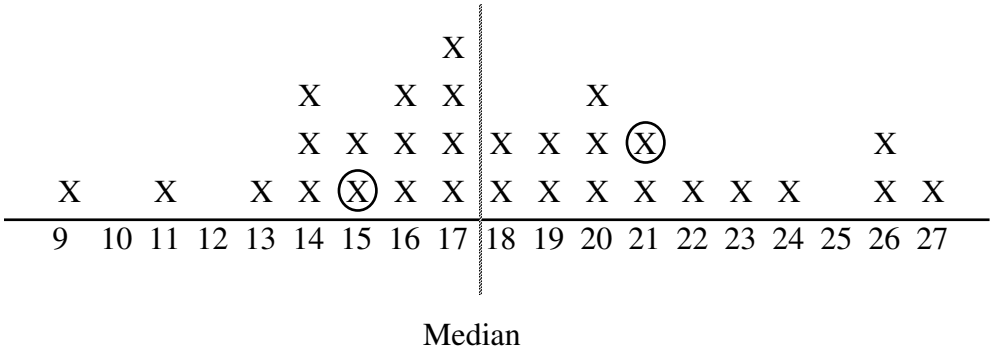
Substituting this value in all three places in the original expression, we get:

$$1\times 81-2\times 81+3\times 81$$

From here we can proceed according to the order of opperations and get 81 – 162 + 243 = **162**.

**2.** A total of 40 people received a total of $2000 in prize money, for a mean (average) payout of $50. Since half the people received only $5, this figure can be misleading. Actually, half the prize money went to one person. The median payout falls halfway between $5 and $10, which is $7.50. The mean was thus $50 – $7.50 = **$42.50** more than the median.

**3.** The median of the data is 17.5, which means that half the data is greater than 17.5 and half the data is less than 17.5. The upper quartile is 21 and the lower quartile is 15. The interquartile range is 21 – 15 = **6**.



Category 5
Algebra
Meet #1, October 2003

**1.** If $A\nabla B$ means $A^2 - 5B$, then find the value of $11\nabla(7\nabla 9)$.

**2.** Find the value of $K$ so that the equation below is an identity. (An identity is an equation that is true for all real values of $x$.)

$$3x - 5(2x + 7) + K = 12x + 3(8 - 7x) + 2x$$

**3.** If $37 = 4x - 3 + 4(3x - 2)$, $y = 24 - 5x$, and $z = 4y + 8$, find the value of $z$.

Answers

1. _____
2. _____
3. _____

# Solutions to Category 5
# Algebra
# Meet #1, October 2003

Answers

**1.** 101

**2.** 59

**3.** 44

**1.** Evaluating $7\nabla 9$ first, according to the rule, we get $7^2 - 5 \bullet 9 = 49 - 45 = 4$. Next we evaluate $11\nabla 4$ and get $11^2 - 5 \bullet 4 = 121 - 20 = \mathbf{101}$

**2.** Simplifying and combining like terms on each side of the equation, we get

$$3x - 5(2x + 7) + K = 12x + 3(8 - 7x) + 2x$$

$$3x - 10x - 35 + K = 12x + 24 - 21x + 2x$$

$$-7x - 35 + K = -7x + 24$$

Adding $7x$ to both sides of the equation, we get:

$$-35 + K = 24$$

Adding 35 to both sides, we find that $K =$ **59**.

**3.** First we must solve the first equation for $x$.

$$37 = 4x - 3 + 4(3x - 2)$$

$$37 = 4x - 3 + 12x - 8$$

$$37 = 16x - 11$$

$$48 = 16x$$

$$x = 3$$

Substituting 3 for $x$ in the second equation, we get:

$$y = 24 - 5 \bullet 3$$

$$y = 24 - 15$$

$$y = 9$$

Finally, we substitute 9 for $y$ in the third equation and get:

$$z = 4 \bullet 9 + 8$$

$$z = 36 + 8$$

$$z = \mathbf{44}$$

Category 6
Team Questions
Meet #1, October 2003

**1.** The Andros children weigh 26 pounds, 52, pounds, 74 pounds, and 92 pounds. The Brooks children weigh 36 pounds and 62 pounds. A math student calculated the average weight of the Andros children and the average weight of the Brooks children. Next he calculated the average of these two averages. What is the positive difference between this average of averages and the actual average weight of the six children?

**2.** The supplement of the complement of angle $B$ measures 34 degrees more than the supplement of angle $B$. How many degrees are in the measure of angle $B$?

**3.** Find the sum of the squares of the proper factors of 42. (The proper factors of a number are all the factors of the number other than the number itself.)

**4.** Evan divided a number by one more than itself and got an amount that was one tenth of his original number. What was Evan's original number?

**5.** In a small village 3/5 of the men are married to 4/7 of the women. (Each married couple is one man and one woman.) What is the smallest possible number of people in the village?

Answers

1. _____ = $A$
2. _____ = $B$
3. _____ = $C$
4. _____ = $D$
5. _____ = $E$
6. _____

**6.** Using the values the team obtained in questions 1 through 5, evaluate the following expression:

$$\sqrt{3\left(\frac{C}{E-D}\right)+\frac{B}{A}}$$

# Solutions to Category 6
# Team Questions
# Meet #1, October 2003

Answers

**1.** 2

**2.** 62

**3.** 736

**4.** 9

**5.** 41

**6.** 10

**1.** The average weight of the four Andros children can be found by adding their weights and dividing by four.

$$\frac{26+52+74+92}{4}=\frac{244}{4}=61 \text{ pounds.}$$

The average weight of the Brooks children is:

$$\frac{36+62}{2}=\frac{98}{2}=49 \text{ pounds.}$$

The average of these two averages (a meaningless figure) is:

$$\frac{61+49}{2}=\frac{110}{2}=55 \text{ pounds.}$$

The actual average weight of the six children is:

$$\frac{(26+52+74+92)+(36+62)}{4+2}=\frac{244+98}{6}=\frac{342}{6}=57 \text{ pounds.}$$

The desired difference is 57 – 55 = **2**.

**2.** The complement of angle $B$ is $90 - B$; the supplement of angle $B$ is $180 - B$; and the supplement of the complement of angle $B$ is $180 - (90 - B)$. Now we can translate the original sentence into the following equation:

$$180-(90-B)=180-B+34$$

Simplifying and solving for B, we get:

$$\begin{aligned} 180-90+B &= 180-B+34 \\ 90+B &= 214-B \\ 90+2B &= 214 \\ 2B &= 124 \\ B &= 62 \end{aligned}$$

There are **62** degrees in the measure of angle B.

**3.** The proper factors of 42 are: 1, 2, 3, 6, 7, 14, and 21. Their squares are 1, 4, 9, 36, 49, 196, and 441 and the sum of these squares is **736**.

**4.** Since the result of Evan's division was one tenth of the original number, he must have divided his original number by 10. Ten is one more than his number, so his original number must be 10 – 1 = **9**.

Algebraically, we could write the equation:

$$\frac{x}{x+1} = \frac{x}{10}$$

Since the numerators are equal, the denominators must also be equal. Thus we have $x + 1 = 10$, so $x = \mathbf{9}$.

**5.** Since every married couple is one man and one woman, it must be true that 3/5 the number of men is equal to 4/7 the number of women. Also, we are counting people, so all values must be whole numbers. This means that the number of men must be a multiple of 5 and the number of women must be a multiple of 7. To get the smallest equal number of married men as married women, we will actually need the Least Common *Numerator* or the LCN. The LCM of 3 and 4 is 12, so the desired ratios are 12/20 of the men and 12/21 of the women. The smallest number of people in the village is thus 20 + 21 = **41**.

**6.** Substituting the values for *A* through *E* into the expression and evaluating, we get:

$$\sqrt{3\left(\frac{736}{41-9}\right)+\frac{62}{2}}$$

$$= \sqrt{3\left(\frac{736}{32}\right)+31}$$

$$= \sqrt{3(23)+31}$$

$$= \sqrt{69+31}$$

$$= \sqrt{100}$$

$$= \mathbf{10}$$