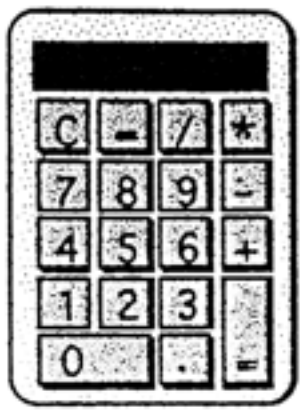
IMLEM 1999-2000



You may use a calculator today!

Category 1 - Mystery
Meet #4 - February, 2000

1) **In an arithmetic sequence, the difference between any two consecutive terms is the same as the difference between any two other consecutive terms. If the 6th term of an arithmetic sequence is 78, and the 9th term is 90, then what is the value of the first term of the sequence ?**

2) **Bob left his home in Peabody, Massachusetts on a Thursday at 1:00 A.M. One million seconds later, he arrived in Miami, Florida. On what day of the week did Bob arrive in Miami ?**

3) **A and B are positive whole numbers.**
A + B = 63.
(A)(B) = 752.
B is larger than A.
What is the value of B ?

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - **Meet #4 - Category 1**

ANSWERS

CATEGORY 1
MYSTERY

1) 58

2) Monday

3) 47

1) To answer this question is equivalent to filling in the first blank in this sequence:

___ ___ ___ ___ ___ 78 ___ ___ 90.

To calculate the difference between two consecutive terms, find the difference between 78 and 90, then divide by 3, as 90 comes three terms after 78:
(90 - 78) ÷ 3 = 12 ÷ 3 = 4.
Subtracting five fours from 78 gives the value of the first term:
78 - (5 x 4) = 78 - 20 = **58**.
To check: The first nine terms, with a difference of four between any two consecutive terms, are
58 62 66 70 74 78 82 86 90.

2) To convert 1,000,000 seconds to days, do the following calculation:

$$\frac{1{,}000{,}000\text{seconds}}{1} \times \frac{1\text{minute}}{60\text{seconds}} \times \frac{1\text{hour}}{60\text{minutes}} \times \frac{1\text{day}}{24\text{hours}}$$

$$= \frac{1{,}000{,}000 \times 1 \times 1 \times 1}{60 \times 60 \times 24}$$

$$= \frac{1{,}000{,}000}{86{,}400}$$

$$= 11.57407\ldots$$

. . . which is a bit more than $11\frac{1}{2}$ days. Eleven days after Thursday is Monday. A bit more than half a day later brings us into the afternoon of that same **Monday**.

3) Guess-and-check works pretty well here. This is a sample solution:

	A	B	A+B	(A)(B)	
Guess #1	20	43	63	860	
Guess #2	18	45	63	810	
Guess #3	16	**47**	63	752	Bingo!

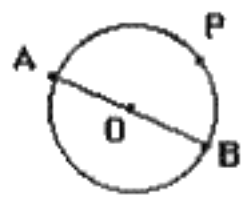
Remember that B must be larger than A.

IMLEM 1999-2000

You may use a calculator today!

Category 2 - Geometry
Meet #4 - February, 2000

1) Let P be any point on the circumference of circle O. $\overline{AB}$ is a diameter. P does not coincide (overlap) with A or B. What is the number of degrees in the measure of angle APB ?



2) The area of a circle is 764.119 square feet. How many feet are in the circumference of the circle ? Use $\pi \approx 3.1$. Round your answer to the nearest tenth.

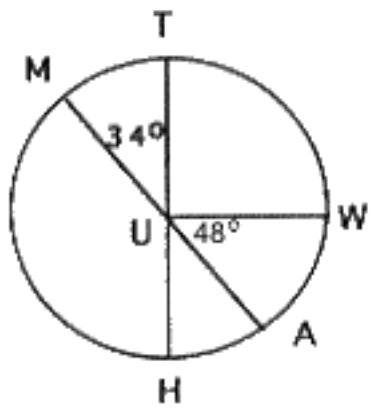
3) $\overline{MA}$ and $\overline{TH}$ are diameters of circle U, as shown below and to the right. TH = 13 centimeters. How many square centimeters are in the area of the sector bounded by radii $\overline{UT}$ and $\overline{UW}$, and minor arc $\overline{TW}$? Round your answer to the nearest tenth. Use $\pi \approx 3.14$.

ANSWERS

1) _____ degrees

2) _____ feet

3) _____ square cm



SOLUTIONS - **Meet #4 - Category 2**

ANSWERS

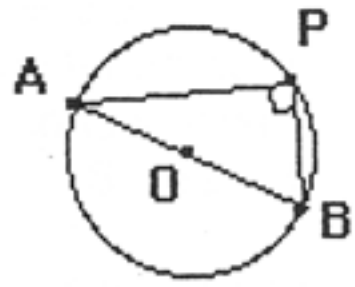
CATEGORY 2
GEOMETRY

1) **90**

2) **97.3**

3) **36.1**

1) It may be simplest to remember that an angle inscribed in a semi-circle is a right angle. Another possibility is that an inscribed angle is half the measure of the intercepted arc. Since angle APB intercepts a semi-circle, or a 180-degree arc, then its measure is half of 180, or 90 degrees. As a third strategy, especially if one is not aware of these theorems, simply draw angle APB, and observe its approximate measure, allowing for the point P to "rove" around the circumference of the circle. Most would notice than the angle APB is always about a right angle!



2)
$$\text{Area} = \pi R^2$$
$$764.119 \approx 3.1R^2$$
$$764.119 \div 3.1 \approx R^2$$
$$246.49 \approx R^2$$
$$\sqrt{246.49} \approx R$$
$$15.7 \approx R$$

If the radius is about 15.7, then the circumference can be found as follows:

$$\text{Circumference} = 2\pi R$$
$$\text{Circumference} \approx (2)(3.1)(15.7)$$
$$\text{Circumference} \approx 97.34$$

Rounded to the nearest tenth, the circumference is about **97.3** feet.

(Next page)

SOLUTIONS - **Meet #4 - Category**

Category 2, continued . . .

3) It helps to know what fraction of the circle is represented by the sector. Knowing the measure of central angle TUW is necessary. Then, the sector represents this fraction of the circle: $\frac{m\angle TUW}{360}$.

$$m\angle TUW + 34 + 48 = 180$$
$$m\angle TUW + 82 = 180$$
$$m\angle TUW + 82 + (-82) = 180 + (-82)$$
$$m\angle TUW = 98$$

So, the sector represents $\frac{98}{360}$ of the circle. Its area is $\frac{98}{360}$ of πR^2, and can be calculated as follows:

$$\left(\frac{98}{360}\right)(\pi R^2)$$
$$= \left(\frac{98}{360}\right)((3.14)(6.5^2))$$
$$= \left(\frac{98}{360}\right)((3.14)(42.25))$$
$$= \left(\frac{98}{360}\right)(132.665)$$
$$= (0.272222...)(132.665)$$
$$= 36.11435...$$

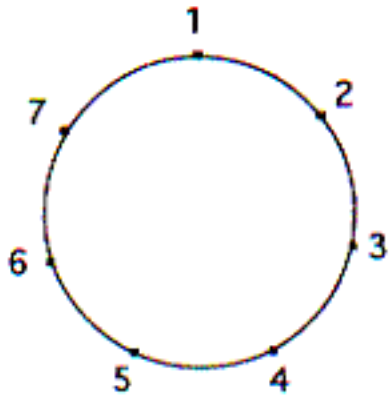
Rounded to the nearest tenth yields the answer **36.1**.

IMLEM 1999-2000

You may use a calculator today!

Category 3 - Number Theory
Meet #4 - February, 2000

1) **A spider spends all of his time walking around the perimeter of a clock like the one shown below. He starts at the point marked 2. Travelling in a clockwise direction, it takes him 14 seconds to arrive for the first time at the point marked 3. If he walks at a constant rate, then which number will he be closest to if his entire journey lasts 6 hours ?**



2) **What is the value of the 913th term of this arithmetic sequence ? Express your answer as a decimal.**

$\frac{4}{5}$ 4.2 $7\frac{3}{5}$ 11 . . .

3) **Bill gave money to his favorite charities according to the following schedule. What was the number of the day just before Bill gave away a total of $10,000,000 ? (Note: After two days, Bill gave away $27,800 + 33,100, or $60,900.)**

ANSWERS

1) _____

2) _____

3) _____

Day #:	1	2	3	4
$ given away:	27,800	33,100	38,400	43,700

SOLUTIONS - **Meet #4 - Category 3**

ANSWERS

CATEGORY 3
NUMBER THEORY

1) 5

2) 3101.6

3) 56

1) Convert 6 hours to seconds, as follows:

$$\frac{6\text{hours}}{1} \times \frac{60\text{minutes}}{1\text{hour}} \times \frac{60\text{seconds}}{1\text{minute}}$$

= (6)(60)(60) seconds

= 21,600 seconds.

The number of "clock-hours" elapsed will be 21,600 ÷ 14, or 1542.85..., which represents 1542.85... ÷ 7 trips around the clock, or 220.408... trips around the clock. It is the remainder, 0.408..., which helps us now. The number of "clock-hours" beyond the starting point, 2, is (o.408...)(7), or 2.85.... Added to the starting point, 2, gives us 2 + 2.85... = 4.85..., which is closest to **5.**

2) The sequence expressed as all decimals is
0.8 4.2 7.6 11.0 . . .
The constant difference is 4.2 - 0.8, or 3.4.
The 913th term is (912)(3.4) + 0.8
= 3100.8 + 0.8
= **3101.6**.

3) This question is equivalent to "Find the sum of the first N terms of the sequence if the sum of those N terms is 10,000,000". The trick is to find the value of N, which can be found through "Guess and check".

(Next page)

SOLUTIONS - **Meet #4 - Category 3**

Category 3, continued . . .

	N	N^{th} term	sum of N terms
Guess #1	10	27,800+10(5300) = 80,800	(27,800+80,800)(10÷2) = 543,000
Guess #2	20	27,800+20(5300) = 133,800	(27,800+133,800)(20÷2) 1,616,000
Guess #3	40	27,800+40(5300) = 239,800	(27,800+239,800)(40÷2) = 5,352,000
Guess #4	60	27,800+60(5300) = 345,800	(27,800+345,800)(60÷2) = 11,208,000
Guess #5	58	27,800+58(5300) = 335,200	(27,800+335,200)(58÷2) = 10,527,000
Guess #6	56	27,800+56(5300) = 324,600	(27,800+324,600)(56÷2) = 9,867,200
Guess #7	57	27,800+57(5300) = 329,900	(27,800+329,900)(57÷2) = 10,194,450

Therefore, the day just before Bill gave away a total of $10,000,000 was the **56**th day.

IMLEM 1999-2000

You may use a calculator today!

Category 4 - Arithmetic
Meet #4 - February, 2000

1) During 1998, Jim's height increased 7%. If he was 62 inches tall at the beginning of the year, then how many inches tall was he at the end of the year ? Round your answer to the nearest tenth.

2) The town of Rockport experienced an 82% growth in its population during the 20th century. If there are now 14,375 people living in Rockport, then how many people were living there at the beginning of the 20th century ? Round to the nearest whole number of people.

3) The dollar value of a Ford Taurus automobile decreases by 25% of its previous year's value. After five years, what is the dollar value of a Ford Taurus whose original value was $17,500 ? Round your answer to the nearest hundredth of a dollar.

ANSWERS

1) _____ Inches

2) _____

3) $ _____

SOLUTIONS - **Meet #4 - Category 4**
February 2000

ANSWERS

CATEGORY 4
ARITHMETIC

1) 66.3

2) 7898

3) 4152.83

1) Let J = the # of inches in Jim's height at the beginning of the year.
Then J + 0.07J = the # of inches in Jim's height at the end of the year.

To answer the question, evaluate this expression:

$$\begin{aligned} & J + 0.08J \\ = \quad & 62 + (0.07)(62) \\ = \quad & 62 + 4.34 \\ = \quad & 66.34 \\ \approx \quad & \mathbf{66.3} \text{ (rounded to the nearest tenth)} \end{aligned}$$

2) Let R = the # of people living in Rockport at the beginning of the century.
Then R + 0.82R = the # of people living in Rockport at the end of the century.

To answer the question, solve this equation:

$$\begin{aligned} R + 0.82R &= 14{,}375 \\ 1R + 0.82R &= 14{,}375 \\ (1 + 0.82)R &= 14{,}375 \\ 1.82R &= 14{,}375 \\ 1.82R \div 1.82 &= 14{,}375 \div 1.82 \\ R &= 7898.351... \end{aligned}$$

Rounded to the nearest whole number, R ≈ **7898**.

(Next page)

SOLUTIONS - **Meet #4 - Category 4**

Category 4, continued . . .

3) original value: $ 17,500

end of year 1: $ 17,500 - (0.25)(17,500)
= 17,500 - 4375
= 13,125

end of year 2: $ 13,125 - (0.25)(13,125)
= 13,125 - 3281.25
= 9843.75

end of year 3: $ 9843.75 - (0.25)(9843.75)
= 9843.75 - 2460.9375
= 7382.812

end of year 4: $ 7382.812 - (0.25)(7382.812)
= 7382.812 - 1845.703
= 5537.109

end of year 5: $ 5537.109 - (0.25)(5537.109)
= 5537.109 - 1384.2772
= 4152.831
≈ $ **4152.83** (rounded)

IMLEM 1999-2000

You may use a calculator today!

Category 5 - Algebra
Meet #4 - February, 2000

1) **Dan the Deli-Man can make six sandwiches in 15 minutes. How many sandwiches can he make in $1\frac{1}{2}$ hours ?**

2) **Gepetto manufactures wooden stools. Some have three legs and the rest have four. He has 161 seats and 551 legs, and wants to use all of the seats and legs. How many of the four-legged stools will Gepetto make ?**

3) **Jake paddles his canoe downstream from Concord to Wayland at a rate of 12 kilometers per hour, and then returns to Concord at six kilometers per hour. If the trip takes a total of four hours, then how many kilometers apart are Concord and Wayland ?**

ANSWERS

1) _____ sandwiches

2) _____ stools

3) _____ kilometers

SOLUTIONS - **Meet #4 - Category 5**

February 2000

ANSWERS

CATEGORY 5

ALGEBRA

1) 36

2) 68

3) 16

1) It helps to make times have the same units - either just minutes or just hours. Minutes are used here, where 1.5 hours = 90 minutes.
Let D = the # of sandwiches made in 90 minutes:

$$\frac{6}{15}=\frac{D}{90}$$

$$15D=(6)(90)$$

$$15D=540$$

$$15D\div 15=540\div 15$$

$$D=\mathbf{36}.$$

2) Let T = the # of 3-legged stools
F = the # of 4-legged stools

Write and solve a system of two equations with two variables:

There are 161 seats: $T + F = 161$
There are 551 legs: $3T + 4F = 551$

Multiply both sides of the first equation by 3:

$3T + 3F = 483$
$3T + 4F = 551$

Then subtract the top equation from the bottom one:

(Next page)

SOLUTIONS - **Meet #4 - Category 5**

Category 5, continued . . .

$3T + 4F = 551$
$3T + 3F = 483$

$F = 551 - 483$
$F = 68$

Therefore, there are **68** four-legged stools.

3) Use the following concept: Rate x Time = Distance.

Let T = the # of hours required to go from Concord to Wayland.
Then 4 - T = the # of hours required to go from Wayland to Concord.

This chart will help to organize information:

	Rate X	Time	= Distance
to Wayland	12	T	12T
to Concord	6	4 - T	6(4 - T)

Since the distance from Concord to Wayland is the same as the distance from Wayland to Concord, we can set the two distances equal to each other:

$12T = 6(4 - T)$
$12T = 6(4) - 6(T)$
$12T = 24 - 6T$
$12T + 6T = 24 - 6T + 6T$
$18T = 24$
$18T \div 18 = 24 \div 18$
$T = 1.3333...$

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SOLUTIONS - **Meet #4 - Category 5**

Category 5, continued . . .

The distance from Concord to Wayland can be calculated as follows:

(rate) (time required)
= (12) (1.333...)
= **16** kilometers.

Check: the return distance, from Wayland back to Concord, is

(rate) (time required)
= (6) (4 - 1.3333...)
= (6) (2.6666...)
= **16** kilometers.

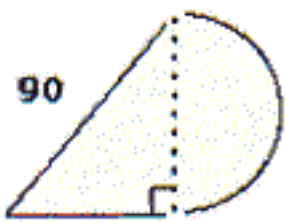
IMLEM 1999-2000

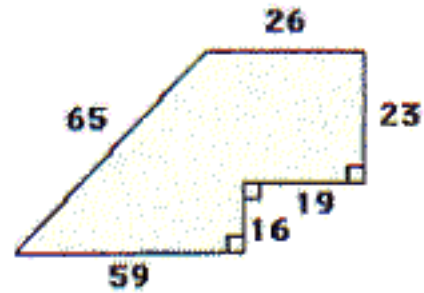
You may use a calculator today!

Category 6 - Team Questions
Meet #4 - February, 2000

1) What is the remainder when 2^{717} is divided by 5 ?

2) This figure below and to the left consists of two line segments and a semicircle whose radius is 36. If the perimeter is $A+B\pi$, then find the sum A + B. (Note: The dotted segment is a diameter of the semicircle.





3) One side of a rectangle is 16 cm, and its area is 144 square cm. How many square cm are in the largest circle which can fit inside the ~~parallelogram~~ rectangle? Use $\pi \approx 3.142$. Round your final answer to the nearest hundredth.

4) How many square units are in the area of the figure to the above-right of #3 ?

5) If $3^X+3^X+3^X+3^X+3^X+3^X=39{,}366$ then find the value of X.

6) Evaluate the expression below, using the answers to questions #1-5 as values for A,B,C,D, and E, respectively:

$$\frac{\left(\sqrt[3]{D+4}\right)\left(\frac{C}{B}\right)\left(\sqrt[4]{3^E}\right)}{A}$$

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

SOLUTIONS - **Meet #4 - Category 6**

ANSWERS

CATEGORY 6
TEAM QUESTIONS

1) 2

2) 180

3) 63.63

4) 1724

5) 8

6) 19.089

1) Calculate the first few powers of 2, then look for patterns:

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$

The pattern of the units digits recurrs in cycles of four:
2 4 8 6 2 4 8 6 2 . . .

The units digit of the 717^{th} power of 2 is 2, because $717 \div 4 = 179$ with a remainder of 1. In other words, 717 is 1 more than a multiple of 4. Every 4th power of 2 has a units digit of 6, and the one following it has a units digit of 2.

If any whole number whose units digit is 2 is divided by 5, then the remainder is **2**.

2) The diameter and the two other segments form a right triangle, and two of its sides are known. Use the Pythagorean Theorem to find the length of the unlabelled segment:

(Next page)

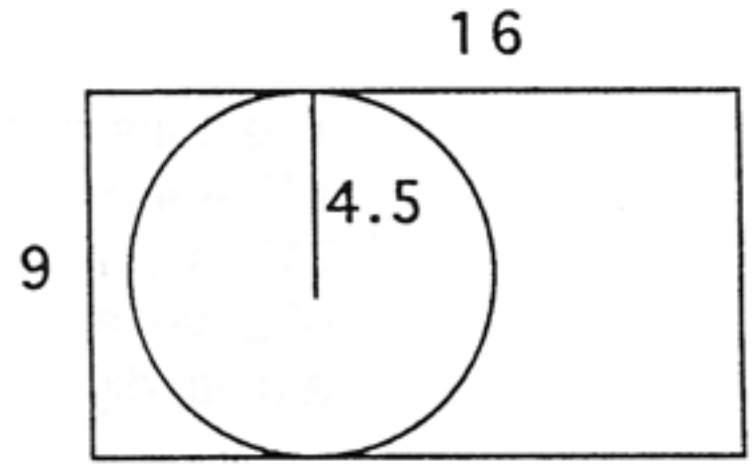
$$A^2 + B^2 = C^2$$
$$72^2 + B^2 = 90^2$$
$$5184 + B^2 = 8100$$
$$5184 + B^2 + (-5184) = 8100 + (-5184)$$
$$B^2 = 2916$$
$$B = \sqrt{2916}$$
$$B = 54$$

The perimeter is equal to $90 + 54 + \pi R$

$= 144 + 36\pi$

Therefore, $A + B = 144 + 36 = \mathbf{180}$.

3) If the area is 144, and one side is 16, then the adjacent side is 144 ÷ 16, or 9.
The largest circle has a diameter equal to the length of the shortest side of the rectangle, as shown below, whose radius is half of 9, or 4.5.



The area of the circle is πR^2

$= (3.142)(4.5^2)$
$= (3.142)(20.25)$
$= 63.6255$
$\approx$ **63.63**. (rounded)

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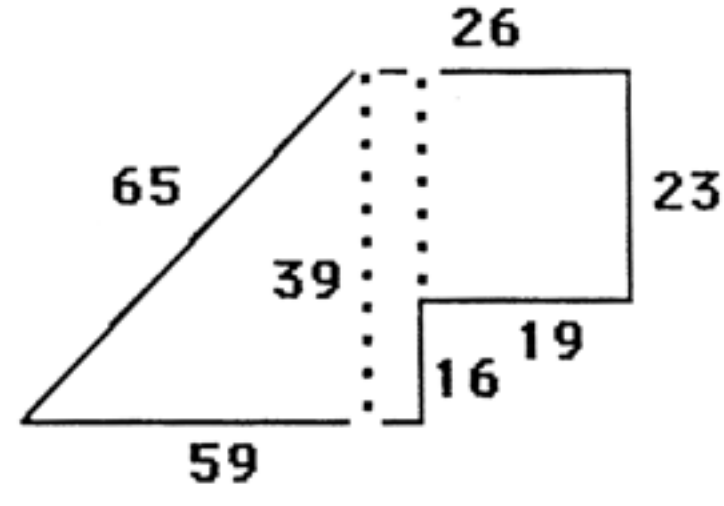
SOLUTIONS - **Meet #4 - Category 6**

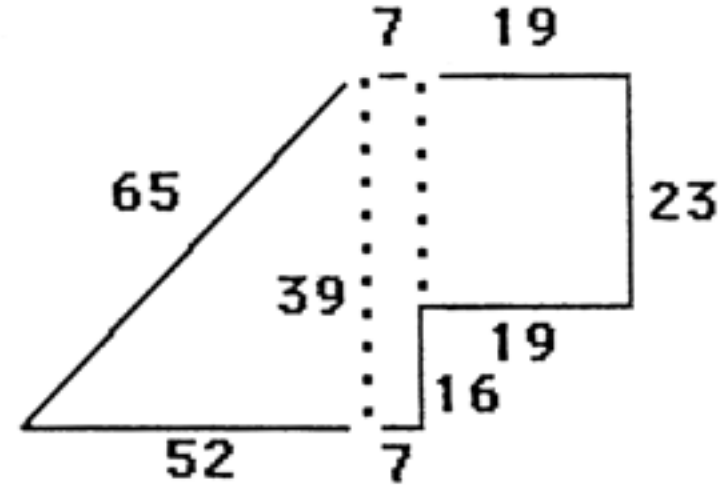
February 2000

Category 6, continued . . . p.3

4) The figure can be subdivided into smaller regions, as shown by the dotted segments:



The length "40" on the top now separates into two parts: 33 and 7. The length "59" on the bottom separates into two parts: 7 and 52, as shown below:



The total area can be found by adding the areas of the three subdivided regions: the triangle and the two rectangles:

Area = triangle + rectangle1 + rectangle 2

$$= \frac{1}{2}BH + LW + LW$$

$$= \frac{1}{2}(52)(39) + (7)(39) + (19)(23)$$

$$= 1014 + 273 + 437$$

= **1724** square units.

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SOLUTIONS - **Meet #4 - Category 6**

February 2000

Category 6, continued . . . p.4

5) $3^X + 3^X + 3^X + 3^X + 3^X + 3^X = 39{,}366$

$$(6)3^X = 39{,}366$$

$$(6)3^X \div 6 = 39{,}366 \div 6$$

$$3^X = 6561$$

Try various powers of 3, through guess-and-check":

$3^5 = 243$
$3^6 = 729$
$3^7 = 2187$
$3^8 = 6561 = 3^x$ Voila! $x=8$

6) $$\frac{\left(\sqrt[3]{D+4}\right)\left(\frac{C}{B}\right)\left(\sqrt[4]{3^E}\right)}{A}$$

$$= \frac{\left(\sqrt[3]{1724+4}\right)\left(\frac{63.63}{180}\right)\left(\sqrt[4]{6561}\right)}{2}$$

$$= \frac{(12)(0.3535)(9)}{2}$$

$$= \frac{38.178}{2}$$

$$= \mathbf{19.089}.$$