IMLEM
1999-2000

Category 1 - Mystery
Meet #3 - January, 2000

1) John has twice as many tapes as George. Paul has seven more than Ringo, but two fewer than John. Ringo has 217 tapes. How many tapes does George have ?

2) Calculate the following: $3\frac{2}{5}(37)+3\frac{2}{5}(16)-3\frac{2}{5}(3)$

3) A plane (a flat surface which extends infinitely) can be divided into two distinct spaces with a single line. Two intersecting lines can divide a plane into four distinct spaces. What is the maximum (most) number of spaces into which a plane can be divided by six lines ?

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #3 - Category 1

ANSWERS

CATEGORY 1
MYSTERY

1) 113

2) 170

3) 22

1) Ringo = 217
Paul = 217 + 7 = 224
John = 224 + 2 = 226
George = 226 ÷ 2 = **113**

2) $3\frac{2}{5}(37)+3\frac{2}{5}(16)-3\frac{2}{5}(3)$

$= 3\frac{2}{5}(37+16-3)$

$= 3\frac{2}{5}(50)$

$= \frac{17}{5}(50)$

$= 17(10)$

$= 170$

3) Count the number of distinct spaces into which a small number of lines can divide the plane, then look for patterns:

# of lines	# of spaces	
1	2	
2	4	2 more than previous
3	7	3 more than previous
4	11	4 more than previous
5	16	5 more than previous
6	22	6 more than previous

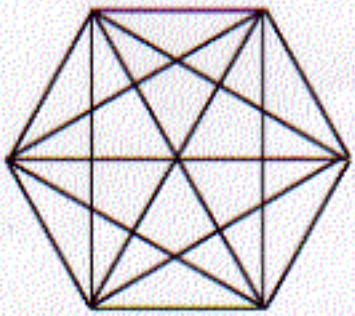
IMLEM
1999-2000

Category 2 - Geometry
Meet #3 - January, 2000

1) How many degrees are in the measure of one interior angle of a regular polygon which has 12 sides ?

2) The diagonal of a parallelogram is perpendicular to the short side. The longer side is 51 inches long, and the short side is 24 inches long. How many square inches are in the area of the parallelogram ?

3) If all the diagonals of a convex hexagon are drawn, as shown in the figure below, the number of diagonals is 9. What is the maximum (most) number of diagonals which can be drawn in a convex polygon which has 24 sides ?



ANSWERS

1) _____ degrees

2) _____ square inches

3) _____

SOLUTIONS - Meet #3 - Category 2

ANSWERS

CATEGORY 2
GEOMETRY

1) **150**

2) **1080**

3) **252**

1) There are at least two ways to solve this problem:
   A. Find the total number of degrees in the 12-gon, then divide by 12, or
   B. Find the measure of one exterior angle, then subtract from 180.

   A. Total = 180 (# of triangles which can be created by drawing all diagonals from one vertex)
   = 180 (# of sides - 2)
   = 180 (12 - 2)
   = 180 (10)
   = 1800
   One interior angle = 1800 ÷ 12
   = **150.**

   B. One exterior angle = 360 ÷ 12
   = 30.
   One interior angle = 180 - 30
   = **150.**

2) The diagonal serves two purposes:
   A. It is one leg of a right triangle, where the lengths of the other two sides are known, and
   B. It is the altitude (height) of the parallelogram, where the short side is the base.

   The Pythagorean Theorem can be used to find the length of the diagonal:

$$A^2 + B^2 = C^2$$
$$24^2 + B^2 = 51^2$$
$$576 + B^2 = 2601$$
$$B^2 = 2601 - 576$$
$$B^2 = 2025$$
$$B = \sqrt{2025}$$
$$B = 45$$

SOLUTIONS - Meet #3 - Category 2

Category 2, continued . . .

An astute observation could also have been made: 24 and 51 are multiples of 3, such that 24 = 3(8), and 51 = 3(17). A Pythagorean triple is 8-15-17. A similar triangle with sides three times as long is 24-**45**-51.

The area of the parallelogram = (base) (height)
= (24) (45)
= **1080**.

3) Start small - count the number of diagonals in polygons of 3, 4, 5 sides, adding one side each time, then look for patterns:

# of sides	# of diagonals	
3	0	
4	2	2 more than previous
5	5	3 more than previous
6	9	4 more than previous
7	14	5 more than previous
8	20	6 more than previous
9	27	7 more than previous
10	35	8 more than previous
11	44	9 more than previous
12	54	10 more than previous
13	65	11 more than previous
14	77	12 more than previous
15	90	13 more than previous
16	104	14 more than previous
17	119	15 more than previous
18	135	16 more than previous
19	152	17 more than previous
20	170	18 more than previous
21	189	19 more than previous
22	209	20 more than previous
23	230	21 more than previous
24	**252**	22 more than previous

IMLEM
1999-2000

Category 3 - Number Theory
Meet #3 - January, 2000

1. The average person breathes about 20,000 times each day. If the average person lives 70 years, then how many breaths does the average person take in a lifetime? Express your answer in scientific notation. (Use 365 days = 1 year.)

2. Express the base 10 numeral **73** as a base 4 numeral. In other words,

$$73_{\text{base 10}} = \underline{\qquad\qquad} \text{ base 4}$$

3. The diameter of a grain of sand is 0.000031 of a centimeter. If grains of sand are placed in a straight line, touching each other, then how many grains of sand are required to measure the length of a corridor (hallway) which is 1240 meters long? Express your answer in scientific notation. (Note: 100 centimeters = 1 meter.)

ANSWERS
1) _____
2) _____
3) _____

SOLUTIONS - Meet #3 - Category 3

ANSWERS

CATEGORY 3
NUMBER THEORY

1) **5.11×10^8**

2) **1021**

3) **4×10^9**

1) # in lifetime = (# in 1 day) (# days in 1 year) (# years in life)
= (20,000) (365) (70)
= 511,000,000
= **5.11×10^8.**

2) 73 base10 = 1(64) + 0(16) + 2(4) + 1(1)
= **1021 base4**

3) # grains = [(1240) (100)] ÷ 0.000031

= 124,000 ÷ 0.000031

$$= \frac{1.24 \times 10^5}{3.1 \times 10^{-5}}$$

$$= \frac{12.4 \times 10^4}{3.1 \times 10^{-5}}$$

$$= \frac{12.4}{3.1} \times \frac{10^4}{10^{-5}}$$

$$= 4 \times 10^{[4-(-5)]}$$

$$= 4 \times 10^{[4+5]}$$

$$= \mathbf{4 \times 10^9}$$

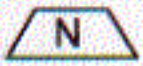
IMLEM
1999-2000

Category 4 - Arithmetic
Meet #3 - January, 2000

1) **Simplify. Express your answer as a mixed numeral in simplest form.**

$$\left(4^2\right)\left(4^{-1}\right)+\left(6^3\right)\left(6^{-5}\right)-\left(3^2\right)\left(3^0\right)$$

2) **Simplify:** $\left[\sqrt[6]{(4\cdot 9)^3}\right]^2$

3) 

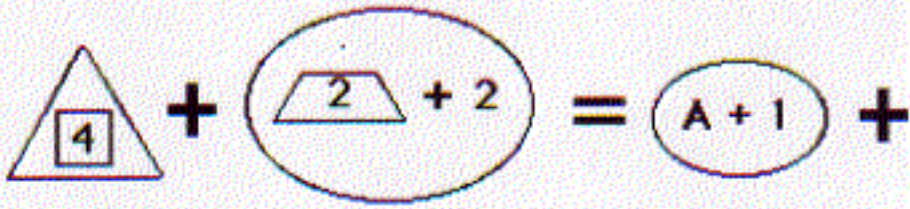
means "Add 3 to N".

means "square the number N".

means "raise N to the 6th power".

means "take the cube root of N".

Find the positive value of A, such that



ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #3 - Category 4

ANSWERS

CATEGORY 4

ARITHMETIC

1) $-4\frac{35}{36}$

2) 36

3) 7

1) $\left(4^2\right)\left(4^{-1}\right)+\left(6^3\right)\left(6^{-5}\right)-\left(3^2\right)\left(3^0\right)$

$= \left(4^{[2+(-1)]}\right)+\left(6^{[3+(-5)]}\right)-\left(3^{[2+(0)]}\right)$

$= \left(4^1\right)+\left(6^{-2}\right)-\left(3^2\right)$

$= 4+\frac{1}{36}-9$

$= 4\frac{1}{36}-9$

$= -4\frac{35}{36}$.

2) $\left[\sqrt[3]{(4\cdot 9)^3}\right]^2$

$= \left[\sqrt[3]{(2^2\cdot 3^2)^3}\right]^2$

(Next page)

Category 4, continued . . .

$$= \left[\sqrt[6]{\left[(2^2)^3(3^2)^3\right]}\right]^2$$

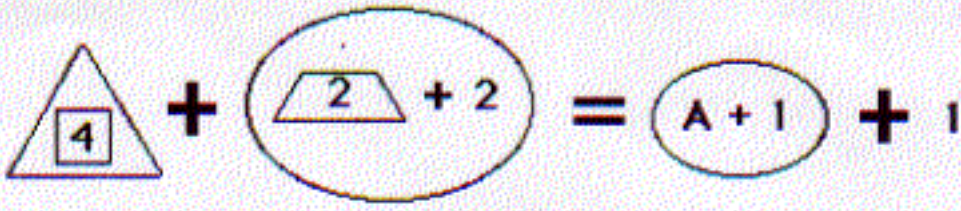
$$= \left[\sqrt[6]{(2^6)(3^6)}\right]^2$$

$$= [(2)(3)]^2$$

$$= [6]^2$$

$$= 36.$$

3)



$$\sqrt[3]{4^6} + [(2+3)+2]^2 = (A+1)^2 + 1$$

$$4^2 + [5+2]^2 = (A+1)^2 + 1$$

$$16 + [7]^2 = (A+1)^2 + 1$$

$$16 + 49 = (A+1)^2 + 1$$

$$65 = (A+1)^2 + 1$$

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IMLEM
1999-2000

Category 5 - Algebra
Meet #3 - January, 2000

1) List the two values of N which make the following equation a true statement:

$$|3N-6|=21$$

2) List the set of all integral (integer) values of W which satisfy the following inequality:

$$-8<5W-4\leq 10$$

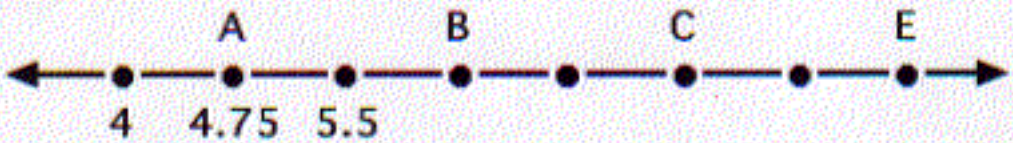
3) The inequality below represents the possible number of hours, H, which Hannah might work in a week:

$$B\leq H\leq C$$

This next inequality represents the number of dollars, D, which she might earn per hour:

$$A\leq D\leq E$$

If the dots are equally spaced, then find the positive difference between her maximum (greatest) possible weekly number of dollars earned, and the minimum (least) possible weekly number of dollars earned.



ANSWERS

1) _____, _____

2) _____

3) $ _____

SOLUTIONS - Meet #3 - Category 5

ANSWERS

CATEGORY 5
ALGEBRA

1) **9** and **-5** (any order)

2) **0, 1, 2** (any order)

3) **42**

1) $|3N-6|=21$

$3N-6=21$ or $3N-6=-21$

$3N-6+6=21+6$ $\quad$ $3N-6+6=-21+6$

$3N=27$ $\quad$ $3N=-15$

$3N\div 3=27\div 3$ $\quad$ $3N\div 3=-15\div 3$

$N=9$ $\quad$ $N=-5$

The two values of N are **9** and **-5**.

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2) $-8<5W-4\leq 10$

$-8+4<5W-4+4\leq 10+4$

$-4<5W\leq 14$

$-4\div 5<5W\div 5\leq 14\div 5$

$-0.8<W\leq 2.8$

The integers which satisfy this inequality are **0**, **1**, and **2**.

---

3) The greatest possible difference can be found as follows:

(greatest possible earnings) - (least possible earnings)

= (C)(E) - (A)(B)

= (7.75)(9.25) - (4.75)(6.25)

= 71.6875 - 29.6875

= **42**.

IMLEM
1999-2000

Category 6 - Team Questions
Meet #3 - January, 2000

1) Regular pentagon MASCO is inscribed in circle T. How many degrees are in the measure of angle ATC if it is less than 180^o ?

2) Simplify. Express your answer as a whole number: $\sqrt[5]{2^2\left(2^7 \cdot 4^5 + 8^3 \cdot 16^2\right)}$

3) Simplify: $\dfrac{4193 \cdot 18 + 6 \cdot 4193}{5 \cdot 4193 + 4193}$

4) N is a 2-digit whole number. What is the largest possible value of N, such that all of the digits of N^2 are non-zero even numbers ?

5) How many different ways can the letters be chosen for the word BIGELOW if the letters must be selected from the configuration below ?

6) Evaluate the expression below, using the answers to #1-5 as values for A,B,C,D, and E, respectively:

$$\frac{\frac{A}{B}(D)(E)\left(\sqrt[4]{BC^2}\right)}{\sqrt{A}(C+E-1)}$$

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

SOLUTIONS - Meet #3 - Category 6

ANSWERS

CATEGORY 6
TEAM QUESTIONS

1) 144

2) 16

3) 4

4) 92

5) 20

6) 240

1) Each central angle of the pentagon measures $360 \div 5$, or 72°. Angle ATC contains two of those angles, $= 2(72)$, or 144°.

2) $\sqrt[5]{2^2\left(2^7 \cdot 4^5 + 8^3 \cdot 16^2\right)}$

$= \sqrt[5]{2^2\left(2^7 \cdot \left(2^2\right)^5 + \left(2^3\right)^3 \cdot \left(2^4\right)^2\right)}$

$= \sqrt[5]{2^2\left(2^7 \cdot 2^{10} + 2^9 \cdot 2^8\right)}$

$= \sqrt[5]{2^2\left(2^{17} + 2^{17}\right)}$

$= \sqrt[5]{2^2\left[2\left(2^{17}\right)\right]}$

$= \sqrt[5]{2^{20}}$

$= 2^4$

$= 16.$

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SOLUTIONS - Meet #3 - Category 6

Category 6, continued . . .

3) $$\frac{4193 \cdot 18 + 6 \cdot 4193}{5 \cdot 4193 + 4193}$$

$$= \frac{4193(18+6)}{4193(5+1)}$$

$$= \frac{4193(24)}{4193(6)}$$

$$= \frac{1(24)}{1(6)}$$

$$= \frac{24}{6}$$

$$= 4.$$

4) Start as high as possible, eliminating as you go along. The square of any odd number will be odd. So, just try the largest two-digit even numbers:

$98^2 = 9604$ no
$96^2 = 9216$ no
$94^2 = 8836$ no
$\mathbf{92}^2 = 8464$ YES!

Answer: **92**.

5) Count them - there are **20**. The following "Pascal's Triangle" type analysis may help, where adding consecutive numbers produces the number below and in between, thus indicating the number of paths which lead to that position in the configuration:

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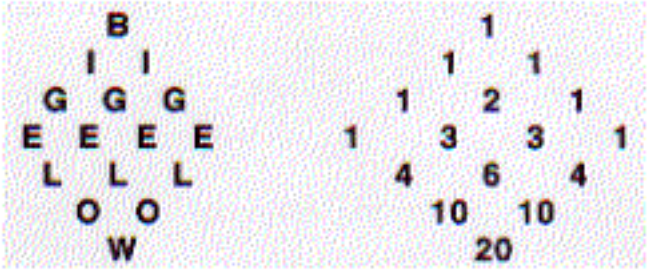
SOLUTIONS - Meet #3 - Category 6

Category 6, continued . . .



6) $$\frac{\frac{A}{B}(D)(E)\left(\sqrt[4]{BC^2}\right)}{\sqrt{A}(C+E-1)}$$

$$= \frac{\frac{144}{16}(92)(20)\left(\sqrt[4]{(16)(4)^2}\right)}{\sqrt{144}(4+20-1)}$$

$$= \frac{(9)(92)(20)\left(\sqrt[4]{(16)(16)}\right)}{12(23)}$$

$$= \frac{(9)(92)(20)\left(\sqrt[4]{4^2 \cdot 4^2}\right)}{12(23)}$$

$$= \frac{(9)(92)(20)\left(\sqrt[4]{4^4}\right)}{12(23)}$$

$$= \frac{(9)(92)(20)(4)}{12(23)}$$

$$= \frac{(3)(3)(4)(23)(20)(4)}{(3)(4)(23)}$$

$$= (3)(20)(4)$$

$$= 240.$$