IMLEM
1999-2000

Category 1 - Mystery
Meet #2 - December, 1999

1) What is the smallest positive whole number which, when divided by 7, leaves a remainder of 1, but when divided by either 3 or 4, leaves a remainder of 2 ?

2) If seven diagonals can be drawn from the vertex of a regular polygon, then how many sides does the polygon have ? (Note: A diagonal is a line segment which connects any two non-consecutive vertices (corner points) of a polygon.)

3) All of the shirts in Veronica's closet are either T-shirts or blouses. Two-fifths of her shirts are blouses. If she were to buy another blouse, then $\frac{3}{7}$ of her shirts would be blouses. How many T-shirts are there in Veronica's closet ?

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #2 - Category 1

ANSWERS

CATEGORY 1
MYSTERY

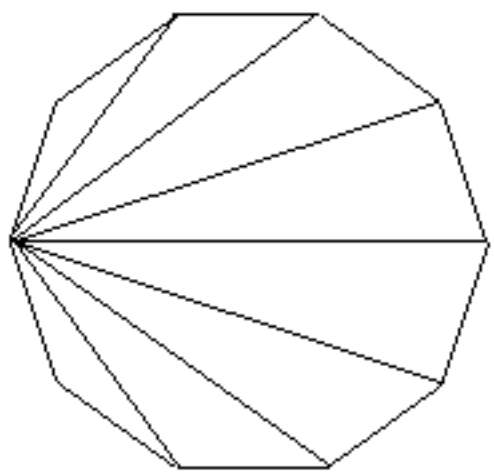
1) 50

2) 10

3) 12

1) We are looking for the smallest positive number which is two more than a multiple of 12, but one more than a multiple of 7. Searching numbers which are two more than multiples of 12 until we reach one more than a multiple of 7: 14 26 38 **50**
Answer: **50**

2) Since a diagonal cannot be drawn from a vertex to either adjacent vertex, or to itself, then the number of sides must be three more than the number of diagonals. Since there are 7 diagonals, there must be 7+3, or **10** sides. Here is a drawing:



3) Suggestion: Create a set of fractions which are equivalent to $\frac{2}{5}$. Then add 1 to both the numerator and denominator until one of the fractions reduces to $\frac{3}{7}$:

$\frac{2}{5}$ $\frac{4}{10}$ $\frac{6}{15}$ $\frac{8}{20}$ . . .

$\frac{3}{6}$ $\frac{5}{11}$ $\frac{7}{16}$ $\frac{9}{21}$ . . .

$\frac{9}{21}$ reduces to $\frac{3}{7}$.

If Veronica had 8 blouses and 12 T-shirts, then $\frac{8}{20}$ of the shirts in her closet would be blouses. Buying one more blouse would make $\frac{9}{21}$ of her shirts be blouses, and $\frac{9}{21}$ reduces to $\frac{3}{7}$.

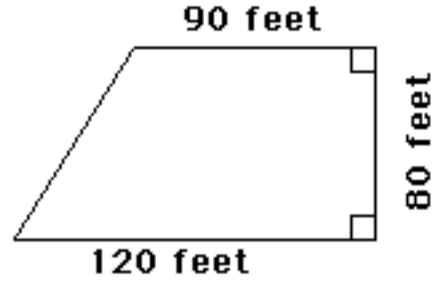
Answer: **12** T-shirts.

MLEM
1999-2000

Category 2 - Geometry
Meet #2 - December, 1999

1) The length and width of a rectangle are positive whole numbers. How many different rectangles are there which have a perimeter of 36 ? (Note: If the length of a rectangle is L, and its width is W, then the LxW rectangle is considered to be the same as the WxL rectangle.)

2) The yard behind Bobby's house is shaped like a trapezoid as pictured below. He has hired someone to seed the yard for a lawn at the rate of 78¢ per ten square feet. How many dollars will Bobby pay? Round your answer to the nearest whole dollar.



3) A square, a regular hexagon, and a regular octagon have equal perimeters. If T = the perimeter of the octagon, and H = half of the perimeter of the hexagon, and the area of the square is 169 square meters, then find the value of T - H.

ANSWERS

1) _____

2) $ _____

3) _____

SOLUTIONS - Meet #2 - Category 2

ANSWERS

CATEGORY 2
GEOMETRY

1) 9

2) 655

3) 26

1) Rectangles with the following dimensions have a perimeter of 36:
1x17 2x16 3x15 4x14 5x13 6x12
7x11 8x10 9x9
Answer: There are **9** rectangles.

2) If Bobby pays 78¢ per **ten** square feet, then he pays 7.8¢ per square foot. His total payment must be
(# of square feet) (# of ¢ per square foot)

$= [\frac{1}{2}h(B+b)](7.8)$

$= \frac{1}{2}(80)(120+90)(7.8)$

$= 40(210)(7.8)$

$= 65{,}520$ ¢

To convert cents to dollars, divide by 100:
$65{,}520 \div 100 = \$\ 655.2$.
Round to the nearest whole dollar: $ **655**.

3) If the area of the square is 169 square meters, then one side is $\sqrt{169}$, or 13 meters, and its perimeter is 4(13), or 52 meters.
T = the perimeter of the octagon = 52 meters.
H = half the perimeter of the hexagon = $\frac{1}{2}(52) = 26$.
T - H = 52 - 26 = **26**.

IMLEM
1999-2000

Category 3 - Number Theory
Meet #2 - December, 1999

1) If $2^A \cdot 3^B \cdot 5^C = 720$,

then find the value of $A^2 + 5B - 4C$

2) At 10:15 A.M., Jared took a sip of water, a sip of orange juice, and a sip of coffee simultaneously (at the same time). If he sips water every 12 minutes, and he sips orange juice every 18 minutes, and coffee every 20 minutes, then at what time of day will he next sip all three drinks simultaneously? (Note: You *must* include A.M. or P.M. with your answer.)

3) The GCF (greatest common factor) of M and P is 8. The LCM (lowest common multiple) of M and P is 120. If M = 40, then what is the value of P ?

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #2 - Category 3

ANSWERS

CATEGORY 3

NUMBER THEORY

1) 22

2) 1:15 P.M.

3) 24

1) If $2^A \cdot 3^B \cdot 5^C = 720$, then working backwards, prime factor the number 720:

720 = 2 x 360
= 2 x 2 x 180
= 2 x 2 x 2 x 90
= 2 x 2 x 2 x 2 x 45
= 2 x 2 x 2 x 2 x 3 x 15
= 2 x 2 x 2 x 2 x 3 x 3 x 5
= $2^4 x 3^2 x 5^1$

Therefore, $A^2 + 5B - 4C$ = $4^2 + 5(2) - 4(1)$
= 16 + 10 - 4
= **22**

2) Find the LCM (lowest common multiple) of 12, 18, and 20:

$12 = 2^2 x 3$

$18 = 2 x 3^2$

$20 = 2^2 x 5$

LCM(12,18,20) = $2^2 x 3^2 x 5$ = 4x9x5 = 180

180 minutes = 3 hours.

Three hours after 10:15 A.M. is **1:15 P.M.**

3) Use the principle that

GCF (M,P) · LCM (M,P) = MP:

8 (120) = 40 (P)

960 = 40 P

960 ÷ 40 = 40 P ÷ 40

24 = P

IMLEM
1999-2000

Category 4 - Arithmetic
Meet #2 - December, 1999

1) Express the following as a mixed numeral in the simplest form $A\frac{B}{C}$, where A, B, and C are whole numbers, and $\frac{B}{C}$ is in lowest terms: $6+\cfrac{6}{6+\cfrac{6}{6+\frac{6}{6}}}$

2) A bag of M&Ms contains 720 pieces of candy. One-fourth of them are red, $\frac{1}{6}$ are yellow, $\frac{1}{18}$ are blue, $\frac{1}{10}$ are tan, and the rest are orange. How many pieces are orange ?

3) Simplify. Write your answer as an improper fraction in lowest terms: $\dfrac{0.\overline{51}+0.\overline{2}}{0.\overline{51}-0.\overline{2}}$

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #2 - Category 4

ANSWERS
CATEGORY 4

ARITHMETIC

1) $6\frac{7}{8}$

2) **308**

3) $\frac{73}{29}$

1) $$6+\frac{6}{6+\frac{6}{6+\frac{6}{6}}}$$

$$= 6+\frac{6}{6+\frac{6}{6+1}} = 6+\frac{6}{6+\frac{6}{7}}$$

$$= 6+\frac{6}{6\frac{6}{7}} = 6+\frac{6}{\frac{48}{7}}$$

$$= 6+\frac{6}{1}\cdot\frac{7}{48}$$

$$= 6+\frac{42}{48}$$

$$= 6+\frac{7}{8}$$

$$= 6\frac{7}{8}$$

2) The fraction of M&Ms which are orange:

$$1-\left(\frac{1}{4}+\frac{1}{6}+\frac{1}{18}+\frac{1}{10}\right)$$

$$= 1-\left(\frac{90}{360}+\frac{60}{360}+\frac{20}{360}+\frac{36}{360}\right)$$

$$= 1-\left(\frac{206}{360}\right)$$

$$= \frac{154}{360}$$

$\frac{1}{360}$ of 720 = 2, so $\frac{154}{360}$ of 720 is 154 (2), or **308**.

3) Converting the repeating decimals to fractions may help:

$$\frac{0.\overline{51}+0.\overline{2}}{0.\overline{51}-0.\overline{2}}$$

$$= \frac{\frac{51}{99}+\frac{2}{9}}{\frac{51}{99}-\frac{2}{9}} = \frac{\frac{51}{99}+\frac{22}{99}}{\frac{51}{99}-\frac{22}{99}}$$

$$= \frac{\frac{73}{99}}{\frac{29}{99}}$$

$$= \frac{73}{99}\div\frac{29}{99} = \frac{73}{99}\cdot\frac{99}{29} = \frac{73}{29}$$

IMLEM
1999-2000

Category 5 - Algebra
Meet #2 - December, 1999

1) The formula for the total surface area of a cone is $\pi r^2 + \pi rh$. Find the number of inches in the height, h, of a cone whose total surface area is 77π square inches, and whose radius, r, is 7 inches.

2) If $3\Psi = \Omega$, and $6 - \vartheta = 9$, and $\Omega + \vartheta = 0$, then find the value of Ψ.

3) Together, Peter and Gordon have 80 goldfish. If Peter gives 20 of his goldfish to Gordon, then Gordon will have 16 more goldfish than Peter. How many goldfish did Gordon have originally ?

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #2 - Category 5

ANSWERS
CATEGORY 5

ALGEBRA

1) 4

2) 1

3) 28

1) Surface Area = $\pi r^2 + \pi rh$

$$77\pi = \pi\left(r^2 + rh\right)$$

$$\frac{77\pi}{\pi} = \frac{\pi\left(r^2 + rh\right)}{\pi}$$

$$77 = r^2 + rh$$

$$77 = (7)^2 + (7)h$$

$$77 = 49 + 7h$$

$$77 + (-49) = 49 + (-49) + 7h$$

$$28 = 7h$$

$$4 = h$$

2) $6 - \emptyset = 9$, so $\emptyset = -3$.
$\Omega + \emptyset = 0$, so $\Omega + (-3) = 0$, and $\Omega = 3$.
$3\Psi = \Omega$, so $3\Psi = 3$, and $\Psi = 1$.

3) The following chart organizes information.
Let X = the # of Peter's original goldfish
80 - X = the # of Gordon's original goldfish

# of Peter's original goldfish	# of Gordon's orignal goldfish	# of Peter's goldfish after he gives 20 to Gordon	# of Gordon's goldfish after he gets 20 from peter
X	80 − X	X − 20	(80 − X) + 20

SOLUTIONS - Meet #2 - Category 5

Category 5 - continued . . .

Using the information in the last two frames, and that Gordon will have 16 more goldfish than Peter, solve this equation:

$$
\begin{aligned}
80 - X + 20 &= X - 20 + 16 \\
100 - X &= X - 4 \\
100 - X + X &= X + X - 4 \\
100 &= 2X - 4 \\
100 + 4 &= 2X - 4 + 4 \\
104 &= 2X \\
104 \div 2 &= 2X \div 2 \\
52 &= X
\end{aligned}
$$

Since $80 - X$ represents the number of goldfish which Gordon had originally,

$$
\begin{aligned}
& 80 - X \\
= \quad & 80 - 52 \\
= \quad & \mathbf{28}.
\end{aligned}
$$

IMLEM
1999-2000

Category 6 - Team Questions
Meet #2 - December, 1999

1) How many three-digit positive prime numbers between 300 and 350 have exactly two digits which are the same ?

2) Roger rode his bicycle 15 miles in 42 minutes. At this rate, how many hours would it take Roger to ride his bicycle from Boston to New York City, if he takes three 25-minute breaks for meals ? The distance along Roger's route is 190 miles. Express your answer as a mixed numeral in simplest form.

3) The length of each side of a parallelogram is multiplied by 6 to create a similar parallelogram whose area is 612 square feet. How many square feet are in the area of the smaller parallelogram ?

4) The heights, in inches, of Moe, Larry, Curly, and Shemp are consecutive odd integers. Shemp is taller than Larry, while Curly is shorter than Larry. Moe is the shortest, and Shemp is the tallest. If the sum of all their heights is 272 inches, then how many inches tall is Larry ?

5) X_W is defined to be $w(w - 1)$. For what value of c does $X_C = 2256$?

6) Evaluate the following expression, using the answers to questions #1-5 as values for A,B,C,D, and E, respectively:

$$\frac{B\left[2D-(C+1)\right]-\left(\frac{E}{A}-1\right)}{D-(E-A)}$$

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

SOLUTIONS - Meet #2 - Category 6

ANSWERS

CATEGORY 6
TEAM QUESTIONS

1) **4**

2) $10\frac{7}{60}$

3) **17**

4) **69**

5) **48**

6) **49**

1) There are two possible searches to conduct -
(1) prime numbers between 300 and 400, and
(2) three-digit numbers between 300 and 400 which have exactly two digits which are the same.

Even numbers can be quickly eliminated, as they are all divisible by 2. Numbers whose units digit is 5 can also quickly be eliminated. The (2) option may be faster, then test each possibility to see if it is prime.

303 311 313 323 331 337 339 343

Again, this list easily elimines numbers which are divisible by 2 or 5.

The following list eliminates (those which are scratched out) multiples of 3 . . .

~~303~~ 311 313 323 331 337 ~~339~~ 343

. . . which leaves us with this list:

311 313 323 331 337 343

The multiple of 7 is shown scratched out below . . .

311 313 323 331 337 ~~343~~

. . . which leaves us with this list:

311 313 323 331 337

SOLUTIONS - Meet #2 - Category 6

Category 6 - continued . . .

None of the remaining numbers is divisible by 11 or 13. However, $323 = 17 \times 19$. All four of the remaining numbers fall below $\sqrt{350}$, or ≈ 18.7. Since none of them is divisible by 17, we can conclude that they are all prime numbers.
Answer: There are **4** such numbers.

2) It helps to convert Roger's speed to miles per hour by solving this proportion:

$$\frac{15}{42} = \frac{X}{60}$$

$$15(60) = 42X$$

$$900 = 42X$$

$$900 \div 42 = 42X \div 42$$

$$21\frac{18}{42} = X$$

$$21\frac{3}{7} = X$$

The total number of hours it should take for Roger to travel from Boston to New York City is

$$\left(190 \div 21\frac{3}{7}\right) + \frac{3(25)}{60}$$

$$= \left(190 \div \frac{150}{7}\right) + \frac{75}{60}$$

$$= \left(190 \cdot \frac{7}{150}\right) + \frac{75}{60}$$

SOLUTIONS - Meet #2 - Category 6

Category 6 - continued . . .

$$= \left(\frac{133}{15}\right) + \frac{75}{60}$$

$$= \left(\frac{532}{60}\right) + \frac{75}{60}$$

$$= \frac{607}{60}$$

$$= \mathbf{10\frac{7}{60}}$$

3) Using the concept that "when an object is enlarged by a scaling factor, its area is enlarged by the square of that scaling factor", we should know that since the smaller parallelogram was scaled by a factor of 6, so its area was scaled by a factor of 6^2, or 36. If the area of the larger parallelogram is 612 square feet, then the smaller one has an area of $612 \div 36$, or **17** square feet.

4) Let X = the number of inches in Moe's height
X+2 = the number of inches in Curly's height
X+4 = the number of inches in Larry's height
X+6 = the number of inches in Shemp's height

$$\begin{aligned} X + (X+2) + (X+4) + (X+6) &= 272 \\ 4X + 12 &= 272 \\ 4X + 12 + (-12) &= 272 + (-12) \\ 4X &= 260 \\ 4X \div 4 &= 260 \div 4 \\ X &= 65 \\ X + 4 &= 69 \end{aligned}$$

Therefore, Larry's height is **69** inches.

SOLUTIONS - Meet #2 - Category 6

Category 6 - continued . . .

5) If $X_W = W(W-1)$,
then $X_C = C(C-1)$.
$2256 = C(C-1)$

No knowledge of quadratics is necessary here, as C and $(C-1)$ must be consecutive integers, whose values can be found through "guessing and checking". $C = 48$, and $(C-1) = 47$.

Therefore, $C =$ **48**.

6) $$\frac{B\left[2D-(C+1)\right]+\left(\frac{E}{A}-1\right)}{D-(E-A)}$$

$$= \frac{10\frac{7}{60}\left[2(69)-(17+1)\right]+\left(\frac{48}{4}-1\right)}{69-(48-4)}$$

$$= \frac{10\frac{7}{60}\left[138-(18)\right]+(12-1)}{69-(44)}$$

$$= \frac{10\frac{7}{60}[120]+(11)}{25}$$

$$= \frac{\frac{607}{60}[120]+(11)}{25}$$

$$= \frac{1214+11}{25}$$

$$= \frac{1225}{25}$$

$$= \mathbf{49}.$$