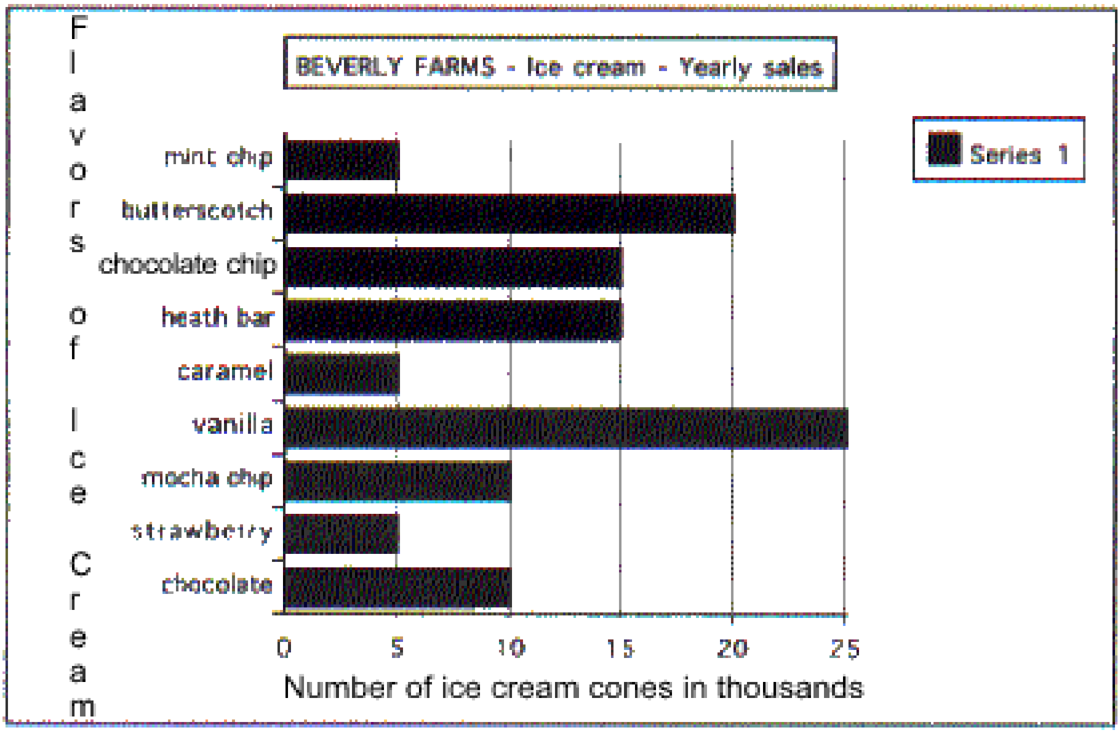
IMLEM
1999-2000

Category 1 - Mystery
Meet #1 - October, 1999

1) **This pattern repeats continuously: M A T H M A T H M A . . .**
What is the 715th letter in the pattern ?

2) **Refer to the bar graph below. The number of vanilla ice cream cones sold was how many more than the number of chocolate chip ice cream cones sold ?**



3) **Willy has a jar containing spiders and queen ants. If he counts just the heads, there are 25. If he counts all of the legs, there are 172. Assuming that each spider has eight legs, and that each ant has six legs, then how many spiders does Willy have in the jar ?**

ANSWERS

1) ____________

2) ____________

3) ____________

SOLUTIONS - Meet #1 - Category 1

October 1999

ANSWERS

CATEGORY 1
MYSTERY

1) **T** **

2) **10,000**

3) **11**

** Problem 1 could also be M if you assume the pattern is the 10-letter sequence MATHMATHMA repeated.

1) Since the pattern repeats in a block of four letters, the letter H occurs at every multiple of four. 715 is 3 more than a multiple of 4. The 712th (a multiple of 4) letter is H. Three letters after that is the letter **T**.

** See note to left

2) Key words - *in thousands.*
Vanilla - 25,000
chocolate chip - 15,000
The difference - **10,000**.

3) A pre-algebra student might use "guess, check, and improve" to solve this problem. The following chart represents one possible approach, starting at a reasonable point:

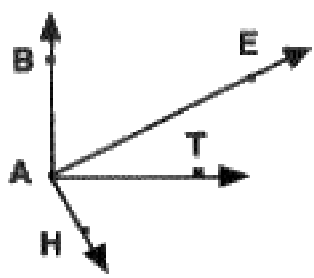
# of spiders	# of ants	# of spider legs	# of ant legs	total # of legs
15	10	120	60	180
14	11	112	66	178
13	12	104	72	176
12	13	96	78	174
11	14	88	84	172

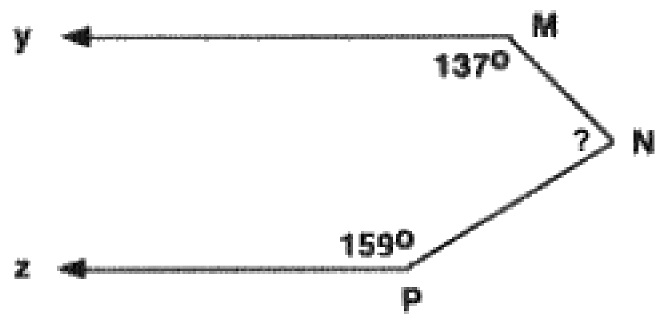
A careful observer may note that if we decrease the number of spiders by one, and therefore increase the number of ants by one, then the total number of legs decreases by two.

IMLEM
1999-2000

Category 2 - Geometry
Meet #1 - October, 1999

1) In the figure to the left below, angle EAT is complementary to angle EAB, and angle EAT is complementary to angle HAT. The measure of angle EAT is 38°. What is the number of degrees in the measure of angle BAH if it is greater than 180° but less than 360° ?





2) In the figure to the right above, lines y and z are parallel. Angle M measures 137° and angle P measures 159°. What is the number of degrees in the measure of angle N if it is less than 180° ?

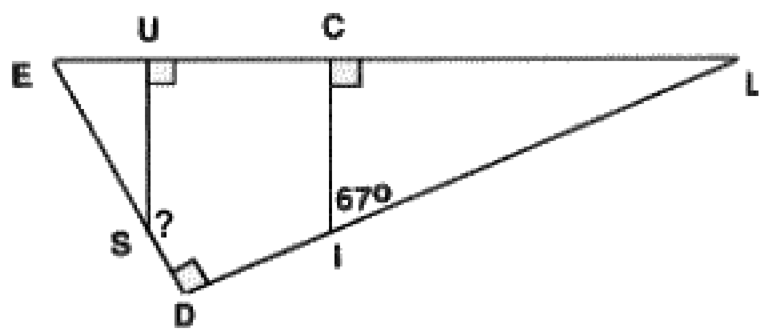
3) In the figure to the right below, the measure of angle CIL is 67°. What is the number of degrees in the measure of angle DSU ?

ANSWERS

1) ______________

2) ______________

3) ______________



SOLUTIONS - Meet #1 - Category 2

October 1999

ANSWERS

CATEGORY 2
GEOMETRY

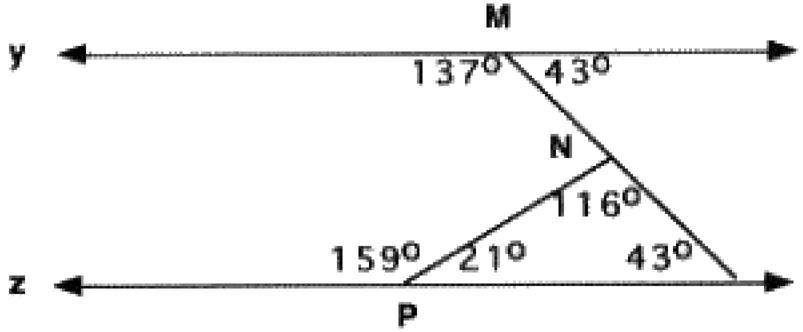
1) 218

2) 64

3) 157

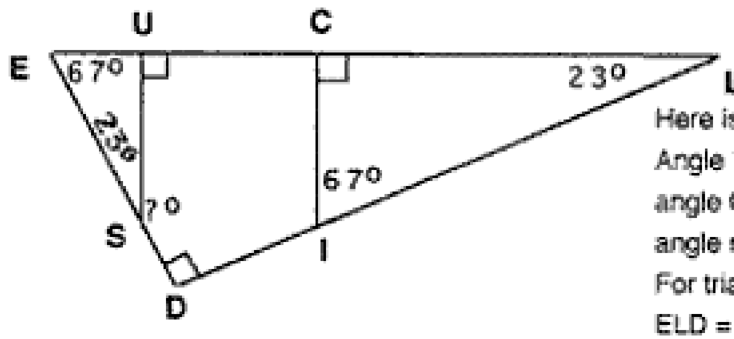
1) $m\angle BAE = 52^o$ (complement of $\angle EAT$)
$m\angle TAH = 52^o$ (also a complement of $\angle EAT$)
$m\angle BAE + m\angle TAH + m\angle EAT + m\angle BAH = 360^o$.
$52 + 52 + 38 + m\angle BAH = 360$
$142 + m\angle BAH = 360$
$m\angle BAH =$ **218^o**.

2) Extend all lines to create a triangle and some pairs of equal angles, and pairs of supplementary angles, as shown below:



Angle measures have been filled in accordingly.
Angle N is the supplement of the 116^o angle, as they are collinear. So, $m\angle N =$ **64^o**.

3) Since there are several right triangles, and the measure of one acute angle is known ($CIL = 67^o$), the measures of several other angles can be found by using the fact that the sum of the measures of any triangle is 180^o.



Here is a possible sequence of thoughts:
Angle ICL = 90^o, angle CIL = 67^o, therefore angle CLI = 23^o, because the sum of the angle measures of any triangle = 180^o.
For triangle EDL: Angle EDL = 90^o, angle ELD = 23^o, therefore angle DEL = 67^o (same reason).
For triangle EUS: Angle EUS = 90^o, angle UES = 67^o, therefore angle ESU = 23^o (same reason).
Angle DSU is the supplement of angle ESU, as they are collinear. Therefore, angle DSU = 180 - 23, or **157^o**.

IMLEM
1999-2000

Category 3 - Number Theory
Meet #1 - October, 1999

1) ☐ is divisible by 3.

   ☐ > 40.

   ☐ is divisible by 5.

   ☐ is not an odd number.

   ☐ < 100.

   ☐ is not divisible by 4.

   What is the value of ☐ ?

2) Two distinct (different) prime numbers are each less than 60. What is the greatest possible product of these two primes ?

3) An abundant number, N, is a whole number whose factors have a sum which is greater than 2N. For example, 12 is an abundant number, because 1+2+3+4+6+12 > 2(12). What is the product of all abundant numbers which are greater than 40 but less than 50 ?

ANSWERS

1) ________

2) ________

3) ________

SOLUTIONS - **Meet #1 - Category 3**

October 1999

ANSWERS

CATEGORY 3

NUMBER THEORY

1) **90**

2) **3127**

3) **2016**

1) Students probably prefer to "filter out" the correct answer by narrowing the field of choices.
Three of the clues narrow the field to a small number of choices - numbers between 40 and 100 which are multiples of 5:

45 50 55 60 65 70 75 80 85 90 95

Those which are divisible by 3:

45 60 75 90

Those which are not divisible by 4:

45 75 90

Finally, the only one left which is not an odd number:

90.

2) The prime numbers yielding the greatest possible product would have to be the largest primes which are less than 60, which are 53 and 59.
Their product is (53)(59), or **3127**.

3) The only numbers between 40 and 50 which have an adequate number of factors to satisfy the conditions forabundant numbers are 42 and 48:
1+2+3+6+7+14+21+42 = 96, which is greater than 2(42).
1+2+3+4+6+8+12+16+24+48 = 124, which is greater than 2(48).
Their product is 42 + 48, or **2016.**

IMLEM
1999-2000

Category 4
Arithmetic
Meet #1 - October, 1999

1) Find the value of the following expression. Round your answer to the nearest tenth.

$$\frac{7+2[6+2\cdot 3^2]}{7\cdot 2-2^3}$$

2) The median of five consecutive prime numbers is 53. What is the mean of the smallest and largest of the five prime numbers ?

3) The arithmetic mean (average) of a set of six numbers is 52. When two numbers, A and B, are added to the set, the arithmetic mean of the new set is 57. What is the arithmetic mean of A and B ?

ANSWERS

1) ________

2) ________

3) ________

SOLUTIONS - Meet #1 - Category 4

ANSWERS

CATEGORY 4

ARITHMETIC

1) **9.2**

2) **52**

1) $\frac{7+2[6+2\cdot 3^2]}{7\cdot 2-2^3}$

$= \frac{7+2[6+2\cdot 9]}{7\cdot 2-8}$

$= \frac{7+2[6+18]}{14-8}$

$= \frac{7+2[24]}{6}$

$= \frac{7+48}{6}$

$= \frac{55}{6}$

$= 9.16666....$

Rounding to the nearest tenth yields the value **9.2**.

2) The number 53 is the middle number (median) of five consecutive prime numbers. So, there are two consecutive prime numbers which are greater than 53, and two which are less. Here is the entire set:

43 47 53 59 61

The mean (average) of the smallest and largest of these numbers is

$\frac{43+61}{2} = \frac{104}{2} =$ **52**.

(Next page)

SOLUTIONS - Meet #1 - Category 4

Category 4 - continued . . .

3) If the average of six numbers is 52, then their sum is 6(52), or 312. If the addition of two more numbers to the set yields an average of 57, then the new sum of the eight numbers is 8(57), or 456.
The number 456 is 144 greater than 312, so the average of A and B must be 144÷2, or 72.

IMLEM

1999-2000

Category 5 - Algebra
Meet #1 - October, 1999

1) $A \$ B$ is defined to be $(A + B)(A - B)$, and
$A ¢ B$ is defined to be $A^2 + B^2$.

What is the value of $(7 \$ 3)(7 ¢ 3)$?

2) Find the value of T which makes the following equation a true statement:

$$4T + 6 + 2(5T - 3) = 39 - 3(2T - 7)$$

3) Find the value of F so that the following equation is an identity, where any value of N produces a true statement:

$$-5 - 2(5 - 4N) = 5N + 3(N - F)$$

ANSWERS

1) ________

2) ________

3) ________

SOLUTIONS - Meet #1 - Category 5

ANSWERS

CATEGORY 5
ALGEBRA

1) 2320

2) 3

3) 5

1) $7 \$ 3 = (7+3)(7-3) = (10)(4) = 40$

$7 ¢ 3 = 7^2 + 3^2 = 49 + 9 = 58$

So, $(7 \$ 3)(7 ¢ 3) = (40)(58) =$ **2320.**

2)

$$4T + 6 + 2(5T - 3) = 39 - 3(2T - 7)$$
$$4T + 6 + 2(5T) - 2(3) = 39 - 3(2T) + 3(7)$$
$$4T + 6 + 10T - 6 = 39 - 6T + 21$$
$$14T = -6T + 60$$
$$14T + 6T = -6T + 6T + 60$$
$$20T = 60$$
$$20T \div 20 = 60 \div 20$$
$$T = \mathbf{3}$$

3)

$$-5 - 2(5 - 4N) = 5N + 3(N - F)$$
$$-5 - 2(5) + 2(4N) = 5N + 3N - 3F$$
$$-5 - 10 + 8N = 8N - 3F$$
$$-15 + 8N = 8N - 3F$$
$$-15 + 8N + (-8N) = 8N + (-8N) - 3F$$
$$-15 = -3F$$
$$-15 \div -3 = -3F \div -3$$
$$\mathbf{5} = F$$

IMLEM 1999-2000

Category 6 - Team Questions
Meet #1 - October, 1999

1) The Maroon Marauders scored a mean of 60 points per game during the 1998 basketball season. They averaged 72 points for each of their 13 home games. How many points did they average for each of their 12 away games ?

2) Moe deLawn cut $\frac{1}{3}$ of the grass in his back yard, while his sister, Sprey, cut $\frac{2}{5}$ of the grass. It then took Moe one hour and 40 minutes to finish cutting the rest of the grass by himself. If Moe had cut all of the grass by himself, then how many hours would it have taken ? Express your answer as a decimal.

3) Two angles are supplementary. If the smaller angle were eight degrees larger, it would still be 14° smaller than its original complement. How many degrees are in the larger of the two original angles ?

4) How many consecutive zeroes are at the end of the product 27 x 28 x 29 x 30 x . . . x 200 ?

5) Eight non-collinear lines intersect on the same plane. What is the maximum possible number of intersection points ?

6) Using the answers to #1-5 as values for A,B,C,D and E, respectively, evaluate the following expression:

$$\frac{3B\left[C-\left(D+3\right)\right]}{2E-\left(A-16\right)}$$

ANSWERS

1) ________ = A

2) ________ = B

3) ________ = C

4) ________ = D

5) ________ = E

6) ________

SOLUTIONS - **Meet #1 - Category 6**

ANSWERS

CATEGORY 6
TEAM QUESTIONS

1) 47

2) 6.25

3) 146

4) 43

5) 28

6) 75

1) For the 25 games, the total score was 25(60), or 1500 points, since the mean of those 25 games was 60 points. If the averrage of the first 13 games was 72 points, then the total of those 13 games was 13(72), or 936 points. The difference between the total of the 25 games and the total of the first 13 games was 1500 - 936, or 564 points, which represents the total of the remaining 12 games. The average of those 12 games is 564 ÷ 12, or **47** points.

2) When Moe finished cutting the grass, he cut $\frac{4}{15}$ of the lawn, because $1 - \left(\frac{1}{3}+\frac{2}{5}\right) = 1 - \left(\frac{5}{15}+\frac{6}{15}\right) = 1 - \left(\frac{11}{15}\right) = \frac{4}{15}$.

If $\frac{4}{15}$ of the lawn required $1\frac{2}{3}$ hours to cut, then the total number of hours required for Moe to cut the entire lawn by himself can be answered by solving this equation, where N represents the total number of hours required by Moe to cut the lawn by himself:

$$\frac{4}{15}N = 1\frac{2}{3}$$

$$N = 1\frac{2}{3} \div \frac{4}{15}$$

$$N = \frac{5}{3} \div \frac{4}{15}$$

$$N = \frac{5}{3} \cdot \frac{15}{4} = \frac{75}{12} = 6\frac{3}{12} = 6\frac{1}{4} = \mathbf{6.25}.$$

(Next page)

Category 6, continued . . .

3) Let X = the smaller angle
Let 90 - X = its complement
Let 180 - X = its supplement

$$X + 8 = 90 - X - 14$$
$$X + X + 8 + (-8) = 90 - X + X - 14 + (-8)$$
$$2X = 68$$
$$2X \div 2 = 68 \div 2$$
$$X = 34$$
$$180 - X = 146.$$

The larger of the two original angles is **146°**.

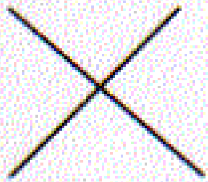
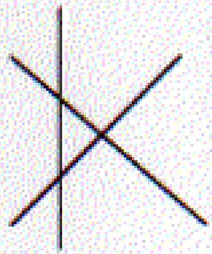
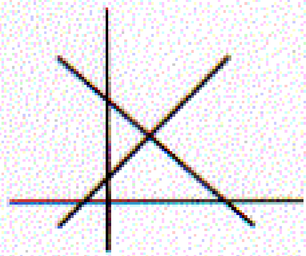
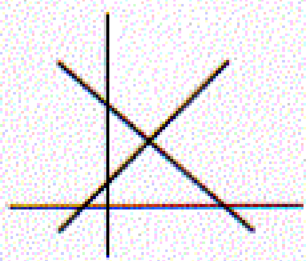
4)

The number of the zeroes are found using the idea that 2(5) = 10. When a number whose units digit is 2 is multiplied by a number whose units digit is 5, the product ends in zero. Therefore, the number of zeroes is determined by how many factors of 5 and how many factors of 2 there are.
From 27 to 200, there are 35 multiples of 5. Some of those contain extra factors of 5, namely 50, 75, 100, 125, 150, 175, and 200. This group of multiples of 5 contains 8 extra multiples of 5, or 43 altogether. There are more than enough factors of 2 from 27 to 200 to be paired with the 43 factors of 5 to produce a total of 43 zeroes at the end of the product..
There are **46** zeroes altogether.

(Next page)

Category 6, continued . . .

5) Solving this problem by actually drawing the lines may be difficult and time-comsuming. Looking for a pattern may help, as follows:

# of lines	maximum number of intersection points	picture
2	1	
3	3	
4	6	
5	10	

Category 6, continued . . .

When an additional line is added to the picture, then one more intersection point is added than was added previously. The number of intersection points forms a sequence called "Triangular numbers", as follows: 1 3 6 10 . . .
If the sequence continues, adding one more than the difference between the previous two terms, then we have: 1 3 6 10 15 21 28.
Therefore, there are **28** points of intersection.

6) $$\frac{3B[C-(D+3)]}{2E-(A-16)}$$

$$= \frac{3(6.25)[146-(43+3)]}{2(28)-(47-16)}$$

$$= \frac{18.75[146-(46)]}{56-(31)}$$

$$= \frac{18.75[100]}{25}$$

$$= \frac{1875}{25}$$

$$= 75.$$