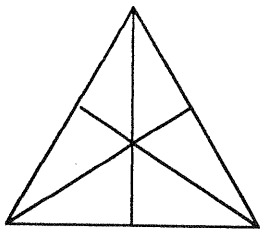
INTERMEDIATE MATHEMATICS LEAGUE OF EASTERN MASSACHUSETTS

MEET #1

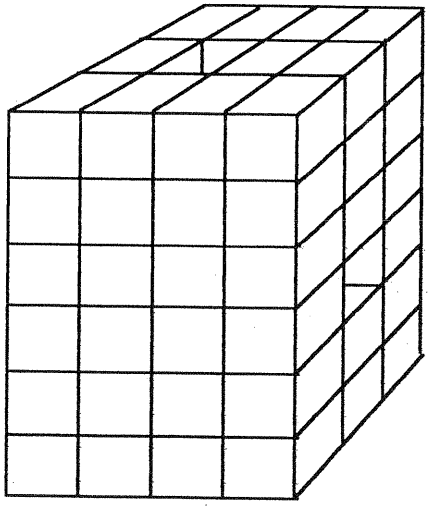
October, 1998

Category 1
Mystery
Meet #1 - October, 1998



1) How many triangles of <u>any</u> size are in this figure?

2) How many of the smallest cubes are there in this figure? Assume that if you see a "hole", then it goes all the way through the figure. If you see as cube, but cannot see what is supporting it from below, then assume that it is supported all the way to the base (bottom) of the figure.



3) Insects have six legs, while spiders have eight legs. Cindy has some insects and spiders in a jar, and counts a total of 34 legs. How many spiders are there?

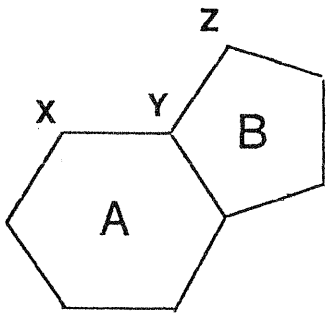
ANSWERS

1) _____

2) _____

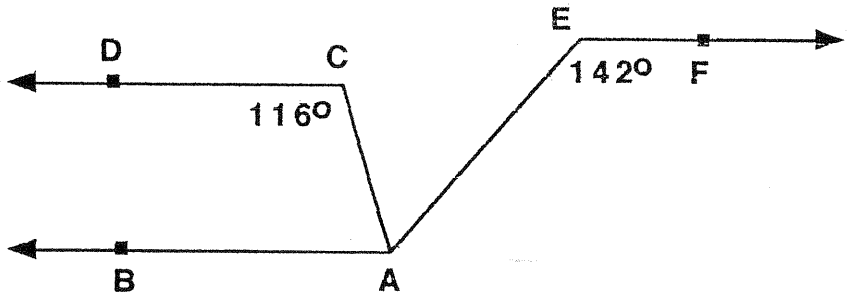
3) _____

Category 2
Geometry
Meet #1 - October, 1998



1) Figure A is a regular hexagon. Figure B is a regular pentagon. What is the measure of angle XYZ, if it is less than 180 degrees?

2) Rays AB, CD, and EF are all parallel to each other. What is the measure of angle CAE. if it is less than 180 degrees?



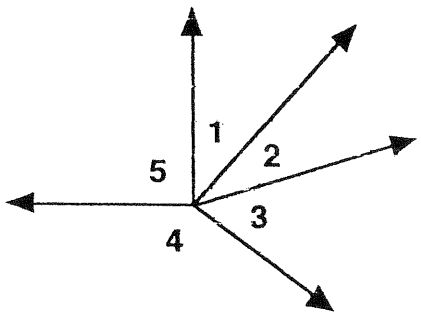
3) In the figure below and to the right, angle 1 is complementary to angle 3. Angle 2 is supplementary to angle 4. Angle 3 is complementary to angle 2. Angle 5 is a right angle. The measure of angle 3 is 52 degrees. How many degrees are in the measure of angle 4?

ANSWERS

1) _____ degrees

2) _____ degrees

3) _____ degrees



Category 3
Number Theory
Meet #1 - October, 1998

1) Find the value of N, if

N is a whole number,
N is divisible by 6,
N > 60,
5 is not a factor of N,
N < 100,
The sum of the digits of N is 12,
N is not divisible by 11.

2) The sum of the composite numbers between 80 and 90 is how much greater than the sum of the prime numbers between 80 and 90? (Do not include the numbers 80 and 90.)

3) 4W25W is a 5-digit whole number which is divisible by 18. What is the value of W?

ANSWERS

1) _____

2) _____

3) _____

Category 4
Arithmetic
Meet #1 - October, 1998

1) Simplify: $7^2 + 4 (10 \times 10 \div 2 \times 5)$

2) Set A = { 9, 2, 9, 7, 4, 9, 6, 7 }
Set B = { 8, 10, 5, 4, 10, 12, 10 }
C = the median of the numbers in set A
D = the mode of the numbers in set B
Find the mean of C and D. Express your answer as a decimal.

3) The mean of four numbers is 12. The mean of five other numbers is 10. What is the mean of all of the nine numbers? Express your answer as a decimal, rounded to the nearest tenth.

ANSWERS

1) _____

2) _____

3) _____

Category 5
Algebra
Meet #1 - October, 1998

1) **Evaluate:** $6\Omega - 4\Delta$
if $\Omega = -5$
and $\Delta = -3.$

2) **Simplify:** $-4N + 3(2N - 5) - 2(4N - 3)$

3) **If** $7 - M = -2$

and $\frac{3}{4} N = 15$

and $-5(7 - E) = -50$

then find the value of H, such that the following equation is an identity:

$$2(M - E) - H = 6N$$

ANSWERS

1) _____

2) _____

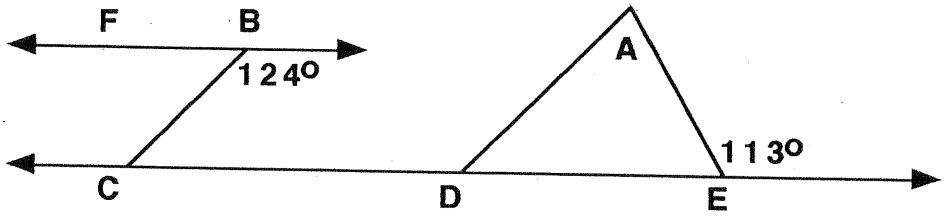
3) _____

Category 6
Team Questions
Meet #1 - October, 1998

1) If A = 4, C = -7, F = 2, N = -5, and W = -8, then evaluate

$$-N^{A} - CW^{F}$$

2) If line FB is parallel to line CE, and segment BC is parallel to segment AD, how many degrees are in the measure of angle A?



3) Find the sum of all prime numbers between 160 and 180.

4) Ted has test scores of 91, 86, 97, and 98. What must be the mean of his next two tests so that the average of all six tests is 90?

5) The supplement of an angle is 14 degrees more than three times its complement. Find the measure of the angle in degrees.

6) Let the answers to problems #1-5 be represented by the letters A, B, C, D, and E. Evaluate the following expression, and give your answer as a decimal.

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

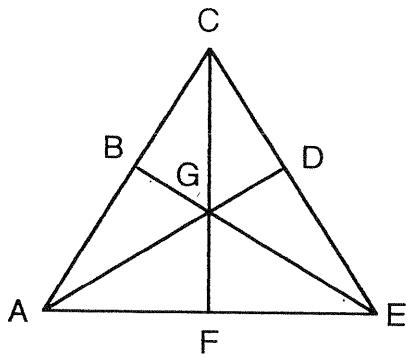
4) _____ = D

5) _____ = E

6) _____

$$\frac{C - B + E}{(B - E)^2} - A + D$$

SOLUTIONS - Meet #1 - Category 1, 1998



ANSWERS

CATEGORY 1
MYSTERY

1) 16

2) 63

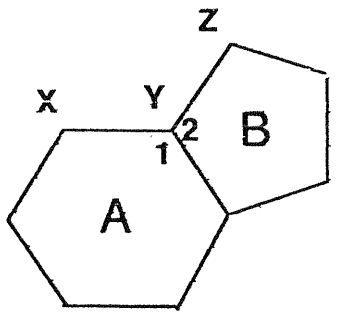
3) 2

1) To keep better track of counting the triangles, the vertices have been lettered.
Here is a complete list of all possible triangles:
BCG, CGD, DGE, GEF, GFA, GBA, ABE, EBC,
ACD, ADE, CFE, CFA, AGC, CGE, AGE, ACE.

2) Beware! The two "tunnels" intersect, so that students may inadvertently count the intersected space twice. If the entire figure were solid, then there would be 6x4x3, or 72 cubes. However, there are six missing in one column, and four missing from a row, for a total of ten missing cubes. Now add one, because the intersected cube was subtracted out twice. Therefore, there are $72 - (6 + 4) + 1$, or **6 3** cubes.

3) Algebra students can be really fascinated by this problem, because it appears that there is only one equation which could represent the facts:
$6X + 8Y = 34$,
where X = the # of insects, and Y = the # of spiders. However, an implied requirement is that X and Y must be whole numbers! Through trial and error, even non-algebra students can quickly garner a solution. There are three insects and two spiders, as $6(3) + 8(2) = 34$. Therefore, there are two spiders.

SOLUTIONS - Meet #1 - Category 2, 1998



ANSWERS

CATEGORY 2
GEOMETRY

1) 132

2) 78

3) 142

1) To calculate the measure of angle XYZ, the sum of angles 1 and 2 must be subtracted from 360.
Angle 1: Since the regular hexagon can be divided into four tringles with diagonals drawn from one vertex, the sum of all those angles = 4(180), or 720. Each interior angle of the hexagon = 720/6, or 120 degrees.
Angle 2: Using the same technique as for the hexagon, the sum of the interior angles of the pentagon is 3(180), or 540. Each interior angle = 540/5, or 108 degrees.
Therefore, the measure of angle XYZ is
360 - (angle 1 + angle 2)
= 360 - (120 + 108)
= 360 - (228)
= **132**

2) When parallel lines or rays are intersected by other lines or rays (transversals), then pairs of angles are created which are either equal or supplementary. Angles DCA and BAC are suppplementary, so that 116 + angle BAC = 180, therefore angle BAC = 64. Angle BAE = angle AEF, so that angle BAE = 142. Since angle BAC + angle CAE = angle BAE, 64 + angle CAE = 142, so angle CAE = **78** degrees.

SOLUTIONS - Meet #1 - Category 2, 1998

Category 2 - continued . . .

3) Angles 2 & 3 are complementary, so angle 2 = 90 - 52, or 38 degrees. Angles 1 & 3 are also complementary, so angle 1 also = 38 degrees.

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360$

$38 + 38 + 52 + \angle 4 + 90 = 360$

$218 + \angle 4 = 360$

$\angle 4 = 142$

Therefore, $\angle 4$ measures **1 4 2** degrees.

SOLUTIONS - Meet #1 - Category 3, 1998

ANSWERS

CATEGORY 3

NUMBER THEORY

1) 84

2) 421

3) 8

1) Listing the multiples of six between 60 and 100 is a reasonable strategy, due to the short size of the list. Numbers from that list can be eliminated by using the other clues, thus leaving the number **84**.

2) Prime numbers between 80 and 90: 83, 89
Composite numbers: 81, 82, 84, 85, 86, 87, 88
(81 + 82 + 84 + 85 + 86 + 87 + 88) - (83 + 89)
= 593 - 172
= **421**

3) For a number to be divisible by 18, it must be divisible by 9 and by 2. Whereas W is a units digit, the only reasonable candidates are 2,4,6,8 or 0. The sum of the five digits must be divisible by 9, so the only possible value for W is **8**, making the sum of the digits 27, which is divisible by 9.

SOLUTIONS - Meet #1 - Category 4, 1998

ANSWERS

CATEGORY 4

ARITHMETIC

1) 1049

2) 8.5

3) 10.9

1) $7^2 + 4(10 \times 10 \div 2 \times 5)$
$= 49 + 4(250)$
$= 49 + 1000$
$= \mathbf{1049}$

There is a tendency for students, while working within the parentheses, to perform the two multiplications first, then to do the division, rather than performing these three *equally-weighted* operations *from left to right!*

2) C = the median of the numbers in set A
= the middle number, when all members are arranged in order, as follows:
2,4,6,**7**,7,9,9
Therefore, C = 7.

D = the mode of the numbers in set B
= the most frequent number in set B
= 10.
Therefore, D = 10.

The mean of C and D is the average of C and D, which is $\frac{C+D}{2} = \frac{7+10}{2} = \frac{17}{2} = \mathbf{8.5}$

3) If the mean of four numbers is 12, then the sum of those four numbers is 4(12), or 48. Likewise, if the mean of the other five numbers is 10, then their sum is 5(10), or 50. So, the mean of all nine numbers is their sum divided by 9, as follows:

$\frac{48+50}{9} = \frac{98}{9} = 10.888...$ which, rounded to the nearest tenth, is **10.9**.

SOLUTIONS - Meet #1 - Category 5, 1998

ANSWERS

CATEGORY 5
ALGEBRA

1) -18

2) -6N - 9
(or -9 - 6N)

3) -96

1) Substitute -5 for Ω and -3 for Δ, then simplify:

$6 \cdot \Omega - 4 \cdot \Delta$
$= 6(-5) - 4(-3)$
$= -30 - (-12)$
$= -30 + 12$
$= \mathbf{-18}$

2) $-4N + 3(2N - 5) - 2(4N - 3)$
$= -4N + 3(2N) - 3(5) - 2(4N) - 2(-3)$
$= -4N + 6N - 15 - 8N + 6$
$= \mathbf{-6N - 9}$

3) First, solve the first three equations to find the values of M, N, and E:
If $7 - M = -2$, then $M = 7 - (-2)$, and **M = 9**.

If $\frac{3}{4} N = 15$, then $N = 15 \div \frac{3}{4}$, and **N = 20**.

If $-5(7 - E) = -50$, then $7 - E = 10$, and **E = -3**.

Now, substitute the values of M, N, and E into the final equation, and find the value of H which will make both members equal:

$2(M - E) - H = 6N$
$2[9 - (-3)] - H = 6(20)$
$2[9 + 3] - H = 120$
$2[12] - H = 120$
$24 - H = 120$
$H = 24 - 120$
$\mathbf{H = -96}$

SOLUTIONS - Meet #1 - Category 6, 1998

ANSWERS

CATEGORY 6

TEAM QUESTIONS

1) -177

2) 57

3) 682

4) 84

5) 52

6) 288.08

1) $-N^A - CW^F$

$= -(-5)^4 - (-7)(-8)^2$

$= -(-5)(-5)(-5)(-5) - (-7)(-8)(-8)$

$= -625 - (-448)$

$= -625 + 448$

$= \mathbf{-177}$

2) Angle BCD = 56 degrees, since it is the supplement of 124 degrees. Angle ADE = 56 degrees, as it corresponds (and is equal) to angle BCD. Angle AED = 67 degrees because it is the supplement of the 113 degree angle. The measures of two angles of triangle ADE are known, so

$$56 + 67 + A = 180$$
$$123 + A = 180$$
$$A = 57$$

Therefore, angle A measures **57** degrees.

3) $163 + 167 + 173 + 179 = \mathbf{682}$

All even numbers between 160 and 180 are composite, because they are each divisible by 2.
161 is not prime, because $161 = 7 \times 23$.
169 is not prime, because $169 = 13 \times 13$.
171 and 177 are each divisible by 3.
165, 170, 175, 180, and 185 are not prime, because each is divisible by 5.

4) If the average of six tests = 90, then the sum of those six scores = 6 (90), or 540. The four known scores have a sum of $91 + 86 + 97 + 98 = 372$. The two unknown scores have a sum of $540 - 372$, or 168, so that their average is $168 \div 2$, or **84**.

SOLUTIONS - Meet #1 - Category 6, 1998

5) Let X = the angle measure
90 - X = its complement
180 - X = its supplement

Now translate the first sentence of the problem into an equation:

$$\begin{aligned} 180 - X &= 3(90 - X) + 14 \\ 180 - X &= 3(90) - 3X + 14 \\ 180 - X &= 270 - 3X + 14 \\ 180 - X &= 284 - 3X \\ 180 - X + 3X &= 284 - 3X + 3X \\ 180 + 2X &= 284 \\ 180 + 2X - 180 &= 284 - 180 \\ 2X &= 104 \\ \frac{2X}{2} &= \frac{104}{2} \\ X &= 52 \end{aligned}$$

Therefore, the angle measures **52** degrees.

6) $\dfrac{C - B + E}{(B - E)^2} - A + D$

$$= \frac{682 - 57 + 52}{(57 - 52)^2} - (-177) + 84$$

$$= \frac{677}{(5)^2} + 177 + 84$$

$$= \frac{677}{25} + 177 + 84$$

$$= 27.08 + 177 + 84$$

$$= \mathbf{288.08}$$