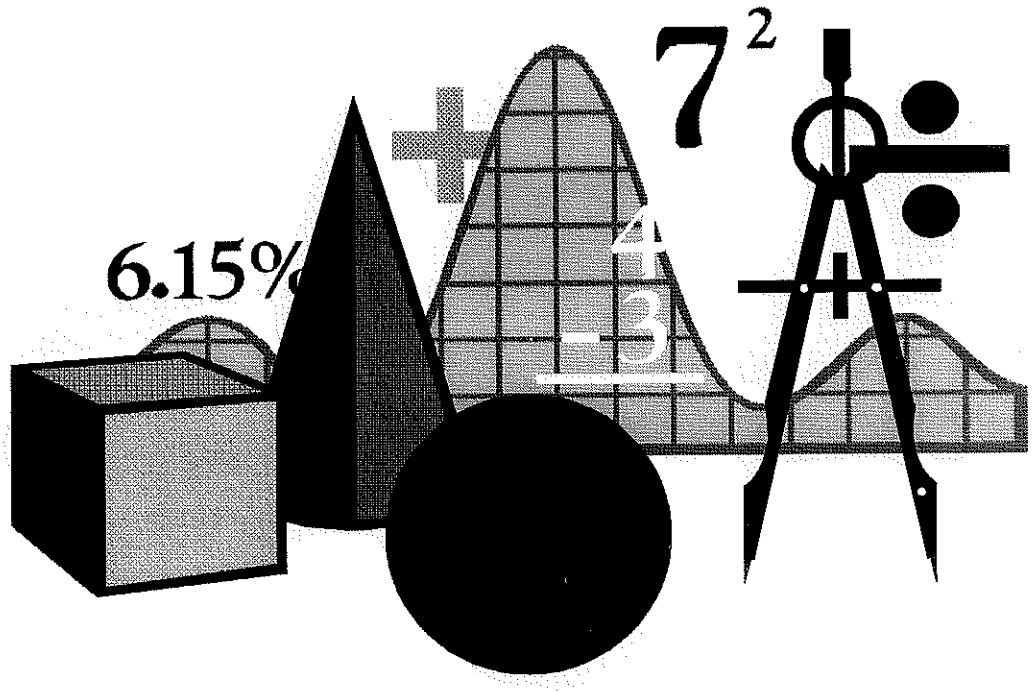
INTERMEDIATE MATH LEAGUE

April 10, 1997

Division 4

MEET #5



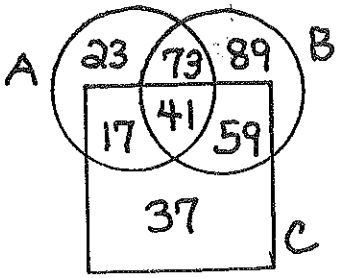
MCCALL MIDDLE SCHOOL

CATEGORY 1 - NUMBER THEORY
APRIL, 1997 - MEET #5

YOU MAY USE A CALCULATOR for all categories today!

① Circles A and B, and Square C intersect to form seven numbered regions. Find the sum of the numbers in the regions formed by the following:

$$(A \cap B) \cup [C \cap (A \cup B)]$$



② Find the mean, or average, of all numbers in $X \cap (Y \cap Z)$
if $X = \{$all integers greater than 724$\}$
$Y = \{$all integers which are divisible by 3$\}$
$Z = \{$all integers less than 745$\}$

③ Of the 2336 students who attend CARBON-8 Middle School,
1431 like Pepsi,
1284 like Coke,
763 like Dr. Pepper,
504 like Coke and Pepsi,
452 like Pepsi and Dr. Pepper,
503 like Dr. Pepper and Coke,
317 like all three drinks
and all of the students like at least one of the drinks.
How many students like only Dr. Pepper?

ANSWERS

① _____

② _____

③ _____

CATEGORY 2 – GEOMETRY
APRIL, 1997 – MEET #5

① A basketball, in the shape of a sphere, is packed inside a closed rectangular box and touches every side of the box. The radius of the basketball is $4\frac{3}{4}$ inches. How many cubic inches of air space is inside the box, but outside the basketball? Use $\pi \approx 3.1$. Round your final answer to the nearest tenth.

② The average hot water tank in a home is in the shape of a cylinder, and contains 40 gallons of water when full. $7\frac{1}{2}$ gallons = 1 cubic foot. If the tank is 42 inches tall, then how many inches are in the diameter of its circular base? Use $\pi \approx 3.14$. Round your final answer to the nearest tenth. (Note: Do no intermediate rounding. In other words, do not round off answers to calculations except for the final calculation.)

③ The floor of a room measures 8 feet by 20 feet, is $12\frac{1}{4}$ feet high, and is rectangular. A spider is on the middle of the 8-foot wall, one foot down from the ceiling, when he discovers that his lunch, a fly caught in a spider web, is on the opposite 8-foot wall, in the middle, and one foot up from the floor. If the spider must travel on the surfaces of the room, then how many feet are in the shortest distance the spider must travel in order to get his lunch? Round your answer to the nearest tenth of a foot.

ANSWERS

① _____ cubic inches

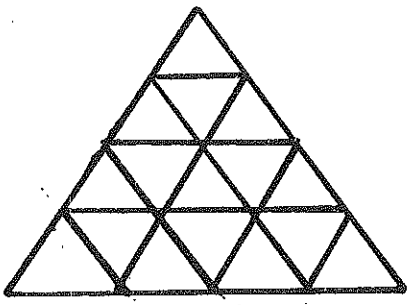
② _____ inches

③ _____ feet

CATEGORY 3 - MYSTERY
APRIL, 1997 - MEET #5

① A boy, who is 5 feet tall, casts a shadow on the ground which is 7.2 feet long. How many feet tall is a telephone pole which casts a shadow 57.6 feet long?

② How many triangles of any size are there in the figure below? (There are more than 16.)



③ Sam can inflate a 6-inch diameter balloon in 8.3 seconds. How many seconds would it take Sam to inflate a balloon which is 15 inches in diameter, if he inflates it at the same rate? Assume that the balloons are spherical (in the shape of a sphere). Round your final answer to the nearest tenth of a second.

ANSWERS

① _____ feet

② _____

③ _____ seconds

CATEGORY 4 – ARITHMETIC
APRIL, 1997 – MEET #5

① In a bag of 468 M+Ms, 84 are red, 92 are yellow, 52 are blue, 114 are tan, 72 are green, and the rest are dark brown. If an M+M is chosen at random from the bag, what is the probability that it is dark brown? Express your answer as a fraction in lowest terms.

② Find the mean (average) of the prime numbers in this list: 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59.

Express your answer as a decimal.

③ Two marbles are selected from a box which contains seven blue and three white marbles. What is the probability that the two marbles selected have the same color? Express your answer as a fraction in lowest terms.

ANSWERS

① _____

② _____

③ _____

CATEGORY 5 – ALGEBRA
APRIL, 1997 – MEET #5

① There are two values of N which make the following equation a true statement. What are those two values?

$$N^2 - 48 = 13N$$

② A rectangle's length is eight inches longer than its width. The area of the rectangle is 425 square inches. How many inches are in the perimeter of the rectangle?

③ An object thrown upward into the air, its distance (d) in meters above where it was thrown is given by the formula $d = rt - 5t^2$ where r is the initial upward velocity (starting speed) and t is the number of seconds since the object was thrown. If a model rocket is launched straight up into the air from the ground, then what is the maximum (highest) height that it reaches if its initial upward velocity is 85 meters per second?

ANSWERS

① { , }

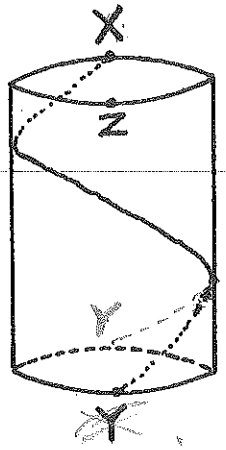
② _____ inches

③ _____ meters

CATEGORY 6 – TEAM QUESTIONS
APRIL, 1997 – MEET #5

① To the nearest day, how many days will elapse during the next million seconds?

② A soup can, in the shape of a right circular cylinder, has a height of $5\frac{1}{2}$ inches. Its circular base has a radius of $1\frac{1}{2}$ inches. Find the length of the shortest distance from point **X** to point **Y** if that distance must be covered on the outside curved surface of the can. Use $\pi \approx 3.14$. Round your final answer to the nearest tenth. Point Y is directly below point Z, which is an endpoint of diameter $\overline{XZ}$.



drawing not to scale

③ A diagonal of a convex polygon is the segment which connects any two of its non-consecutive vertices. How many diagonals does a polygon of 24 sides have?

④ If ☺ # ☹ = $☹^2 - ☺^2$
and $\triangle \not\subset \square = (\triangle + \square)^2$
then find the value of $(2 \not\subset 5) \# 12$.

⑤ Find the sum of the two values of W which solve the following equation:

$$W^2 + 5W + 21 = 69 - 3W$$

⑥ If A, B, C, D, and E represent the answers to #1-5 above, respectively, then evaluate:

$$\sqrt{\frac{A}{4} + C - D - 10B - E - 10}$$

ANSWERS

① _____ = A

② _____ = B

③ _____ = C

④ _____ = D

⑤ _____ = E

⑥ _____

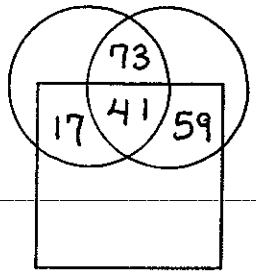
SOLUTION KEY
APRIL, 1997

CATEGORY 1

① 190

② 735

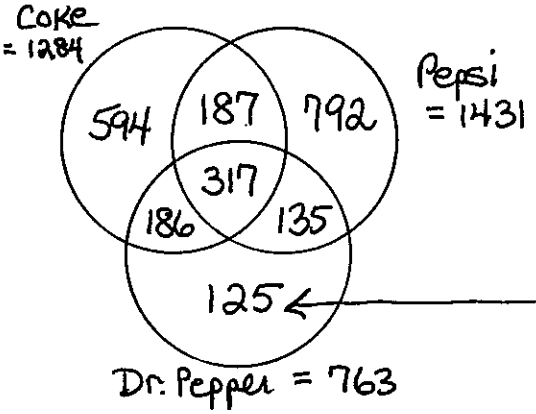
③ 125

① 

The region formed by the required unions and intersections contains the numbers listed at the left. The sum of the numbers is $73 + 17 + 41 + 59 = 190$

② The numbers belonging to $X \cap (Y \cap Z)$ are the multiples of 3 which are between 724 and 745, which are 726, 729, 732, 735, 738, 741, and 745. Their average is equal to $\frac{\text{their sum}}{7}$, or 735. An astute observer may notice that, due to the numerical symmetry of the set, and that there is an odd number of numbers, the mean is also the median : 735!

③ The following Venn diagram of three intersecting circles may help. All of the given clues imply the numbers listed in the various regions.



∴ The number who like only Dr. Pepper is 125.

CATEGORY 2

① 414.4

② 16.7

③ 29.8

① Volume of air space = Volume of box − Volume of ball

$= l \cdot w \cdot h - \frac{4}{3}\pi r^3$

$= 9\frac{1}{2} \cdot 9\frac{1}{2} \cdot 9\frac{1}{2} - \frac{4}{3}\pi(4\frac{3}{4})^3$

$= 857.375 - \frac{4}{3}\pi(107.171875)$

$\approx 857.375 - 442.97703$

≈ 414.3979

≈ 414.4 (nearest tenth)

SOLUTION KEY – continued...

Category 2, continued...

② Volume of tank, in cubic feet:

$$\frac{40 \text{ gal.}}{\text{tank}} \times \frac{1 \text{ cu.ft.}}{7\frac{1}{2} \text{ gal.}} = \frac{40}{7\frac{1}{2}} \frac{\text{cu.ft.}}{\text{tank}} = 5\frac{1}{3} \text{ cu.ft.}$$

Volume of any cylinder $= \pi r^2 h$

Volume of tank $= \pi r^2 \cdot 3\frac{1}{2} = 5\frac{1}{3}$ $\quad (42'' = 3\frac{1}{2}')$

$(3.14)(r^2)(3.5) = 5\frac{1}{3}$

$10.99\, r^2 = 5\frac{1}{3}$

$r^2 = 5\frac{1}{3} \div 10.99$

$r^2 \approx 0.4852896\ldots$

$r \approx \sqrt{0.4852896\ldots}$

$r \approx 0.69662734\ldots$

$\therefore$ diameter $= 2r \approx 1.3932547\ldots$ feet

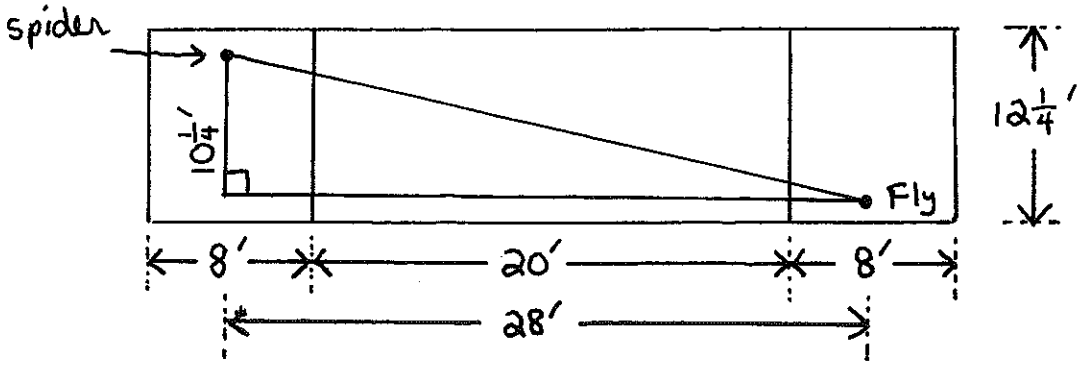
Now convert to inches: $12(1.393257\ldots)$

$\approx 16.719056\ldots$

≈ 16.7 inches

③ If the spider climbs down the wall ($11\frac{1}{4}'$), across the floor ($20'$), then up the opposite wall to the fly ($1'$), he travels a total of $32\frac{1}{4}$ feet.

HOWEVER, there is a shorter distance, if the spider walks diagonally across the walls, simulating the hypotenuse of a right triangle. Folding out three walls so they lie flat:



SOLUTION KEY, continued...

category 2, #3 continued...

Use the Pythagorean Theorem to find the length of the hypotenuse, which is the shortest path from the spider to the fly:

$$a^2 + b^2 = c^2$$
$$(10\tfrac{1}{4})^2 + (28)^2 = c^2$$
$$105.0625 + 784 = c^2$$
$$889.0625 = c^2$$
$$\sqrt{889.0625} = c$$
$$29.82 \approx c$$
$$29.8 \approx c$$

∴ The shortest distance, rounded to the nearest tenth of a foot, is 29.8 feet!

CATEGORY 3

① 40

② 27

③ 129.7

① Use a proportion to solve: Let x = height of the telephone pole

$$\frac{\text{height of object}}{\text{length of shadow}} \qquad \frac{5}{7.2} = \frac{x}{57.6}$$

$$7.2x = 5(57.6)$$
$$7.2x = 288$$
$$x = 288 \div 7.2$$
$$x = 40$$

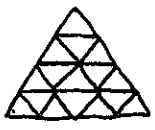
∴ The telephone pole is 40 feet tall.

② △ = 16

 = 6

 = 3

 = 1

 = 1

TOTAL = 27

Category 3, continued...

③ The larger balloon has a diameter which is 2.5 times as long as the smaller balloon ($15 \div 6 = 2.5$). So the larger balloon has 2.5^3, or 15.625 times as much volume, requiring 15.625 times as much time to inflate it.

$\therefore \quad 15.625(8.3) = 129.6875$
≈ 129.7 (rounded to the nearest tenth).

A student may calculate the actual volumes of the balloons to arrive at the same answer:

Volume of larger balloon: $\frac{4}{3}\pi \cdot 15^3$
Volume of smaller balloon: $\frac{4}{3}\pi \cdot 6^3$

Divide to find the ratio of volumes:

$$\frac{\frac{4}{3}\pi \cdot 3375}{\frac{4}{3}\pi \cdot 216}$$

$= 15.625$

Then $15.625(8.3) = 129.6875$
≈ 129.7

CATEGORY 4

① $\frac{3}{26}$

② 48.6 (must be a decimal)

③ $\frac{8}{15}$

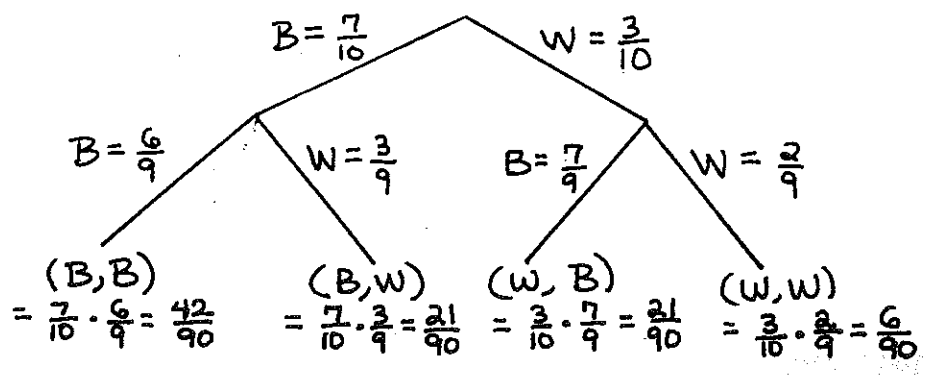
① # of dark brown $= 468 - (84 + 92 + 52 + 114 + 72)$
$= 468 - 414$
$= 54$

The probability that a dark brown M+M is chosen is $\frac{54}{468} = \frac{27}{234} = \frac{3}{26}$

② The average of the PRIME numbers in the list is

$$\frac{41 + 43 + 47 + 53 + 59}{5} = \frac{243}{5} = 48.6$$

③



Category 4, continued...

For the two marbles to have the same color, they both must be blue or both must be white:

(B,B) or $(W,W) = \frac{42}{90} + \frac{6}{90} = \frac{48}{90} = \frac{24}{45} = \frac{8}{15}$

CATEGORY 5

① $-3, 16$ (any order)

② 84

③ 361.25 or $361\frac{1}{4}$

① $N^2 - 48 = 13N$

$N^2 - 13N - 48 = 0$

$(N-16)(N+3) = 0$

$N - 16 = 0$ or $N + 3 = 0$

$N = 16$ or $N = -3$

② 

$x(x+8) = 425$

$x^2 + 8x = 425$

$x^2 + 8x - 425 = 0$

$(x+25)(x-17) = 0$

$x + 25 = 0$ or $x - 17 = 0$

$x = -25$ or $x = 17$ ← width

(cast out, as lengths cannot be negative.)

$x + 8 = 25$ (length)

Perimeter $= 2 \cdot \text{length} + 2 \cdot \text{width}$

$= 2 \cdot 25 + 2 \cdot 17$

$= 50 + 34$

$= 84$

③ The maximum height is reached halfway between when the rocket is launched, and when it lands. The time it takes to land is when $d = 0$:

$d = rt - 5t^2$

$0 = 85t - 5t^2$

$0 = 5t(17 - t)$

$t = 0$ (time at launch)

or $t = 17$ (time at landing)

April 1997

SOLUTION KEY, continued ...

Category 5, question #3 continued...

∴ It takes half of 17 seconds, or 8.5 seconds, for the rocket to reach its maximum height. Now substitute 8.5 for t into the formula:

$$d = rt - 5t^2$$
$$d = 85(8.5) - 5(8.5)^2$$
$$d = 722.5 - 361.25$$
$$d = 361.25$$

CATEGORY 6

① 12

② 10.9

③ **252**

④ −2257
(watch for the negative sign!)

⑤ −8

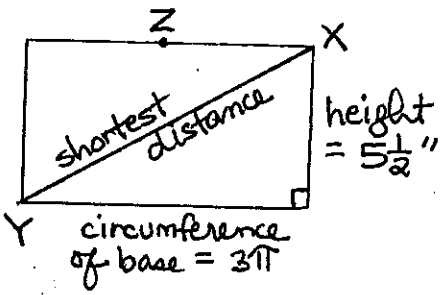
⑥ 49

① $\frac{1{,}000{,}000 \text{ sec.}}{1} \times \frac{1 \text{ min.}}{60 \text{ sec.}} \times \frac{1 \text{ hr.}}{60 \text{ min.}} \times \frac{1 \text{ day}}{24 \text{ hr.}}$

$= \frac{1{,}000{,}000}{60 \times 60 \times 24}$ days $= 11.57407\ldots$

≈ 12 (rounded to the nearest day)

② Imagine cutting a soup can label from a can, starting at point X and cutting straight down to the base. Unroll the label. It should look like this:



Use the Pythagorean Theorem:

$$\left(5\tfrac{1}{2}\right)^2 + (3\pi)^2 = (XY)^2$$
$$30.25 + (3 \cdot 3.14)^2 \doteq (XY)^2$$
$$30.25 + 88.7364 \doteq (XY)^2$$
$$118.9864 \doteq (XY)^2$$
$$\sqrt{118.9864} \doteq XY$$
$$10.908\ldots \doteq XY$$
(nearest tenth) $10.9 \doteq XY$

③ A possible strategy: Look for a pattern in diagonals for polygons of an increasing number of sides:

# of sides	# of diagonals	
3	0	
4	2	+2
5	5	+3
6	9	+4
7	14	+5
8	20	+6
⋮	⋮	

SOLUTION KEY, continued ...

Category 6, continued ...

④ $(2 \text{¢} 5) \# 12$

First, $2 \text{¢} 5 = (2+5)^2$
$= 7^2$
$= 49$

Then $49 \# 12 = 12^2 - 49^2$
$= 144 - 2401$
$= -2257$

⑤ $W^2 + 5W + 21 = 69 - 3W$

$W^2 + 5W + 3W + 21 - 69 = 0$

$W^2 + 8W - 48 = 0$

$(W+12)(W-4) = 0$

$W+12 = 0$ or $W-4 = 0$

$W = -12$ or $W = 4$

∴ The sum of the values of W is $(-12) + 4 = -8$.

⑥ $\sqrt{\frac{A}{4} + C - D - 10B - E - 10}$

$= \sqrt{\frac{12}{4} + 252 - (-2257) - 10(10.9) - (-8) - 10}$

$= \sqrt{3 + 252 + 2257 - 109 + 8 - 10}$

$= \sqrt{2401}$

$= 49$

I hope you all enjoyed this year's contests!

☺ Vaag