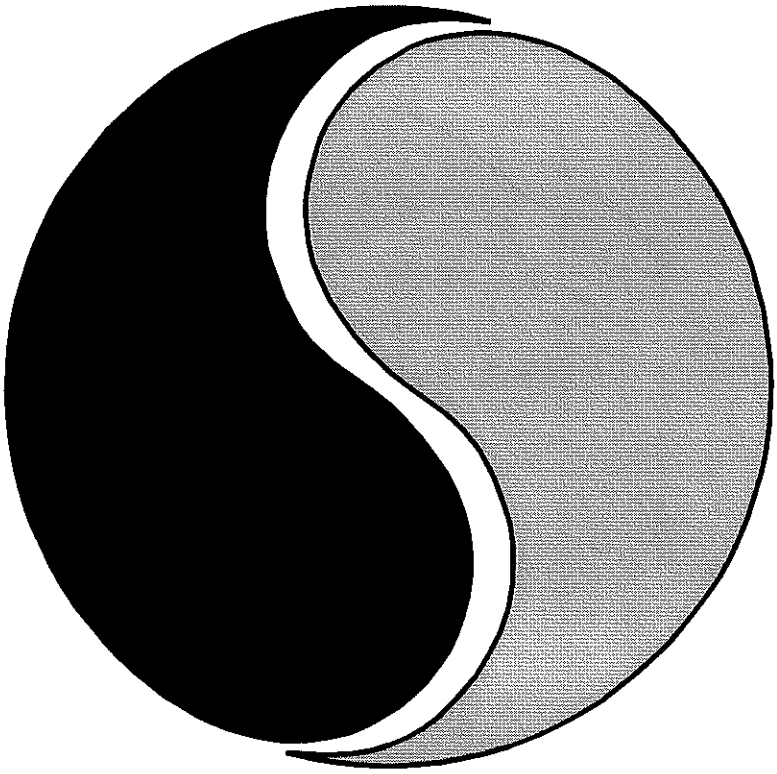
INTERMEDIATE MATH LEAGUE

March 6, 1997

Division 4

MEET #4



MCCALL MIDDLE SCHOOL

CATEGORY 1 – NUMBER THEORY
March 1, 1997 – Meet #4

YOU MAY USE A CALCULATOR FOR ALL CATEGORIES TODAY.

① Find the value of the 12^{th} term in this sequence:

51.2 76.8 115.2 172.8 259.2 ...

Express your answer as a decimal.

② Willy gave away chocolate bars every hour, on the hour, according to the following pattern:

Date and Time	# of chocolate bars
Mon., March 3, 7:00 A.M.	4
Mon, March 3, 8:00 A.M.	10
Mon, March 3, 9:00 A.M.	16
Mon., March 3, 10:00 A.M.	22
⋮	⋮

If the last chocolate bars were given away at 4:00 P.M. on Thursday, March 6, then how many chocolate bars did Willy give away in all from Mon, March 3 at 7:00 A.M. through Thursday, March 6 at 4:00 P.M.?

③ The numbers of students at Alpha Beta Middle School is somewhere between 880 and 900. On the first day of school, all of the students lined up in a straight line, and were assigned to homerooms based upon the letter of the alphabet assigned to them. The first five students in line were assigned the letters A, B, C, D, E, and so on. When the letter Z was reached, the pattern cycled back to A, B, C, etc. The last student in line said "H". How many students are there at Alpha Beta?

ANSWERS

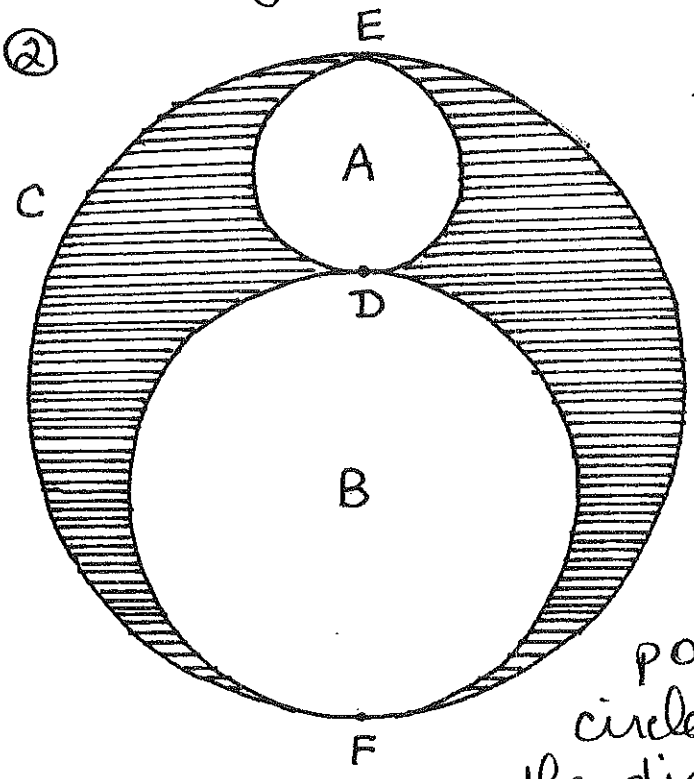
① _____

② _____

③ _____

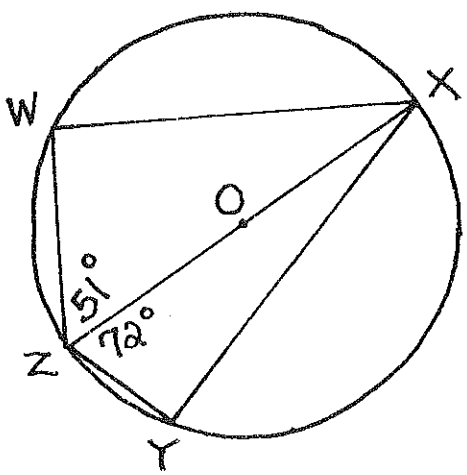
CATEGORY 2 - GEOMETRY
MARCH, 1997 - MEET #4

① Find the area, in square inches, of the largest circle which can be drawn on a sheet of paper measuring $8\frac{1}{2}$ inches by 11 inches. Use $\pi \approx 3.1$. Round your final answer to the nearest thousandth.

② 

In the figure at the left, Circle A is tangent to circle C at point E, and is tangent to circle B at point D. Circle B is tangent to Circle C at point F. Two circles are tangent if their intersection is only one point. The diameter of circle B is twice as long as the diameter of circle A. The diameter of circle C is 18 cm. Using $\pi \approx 3.14$, what is the area of the shaded region, in square centimeters?

③ In the figure at the right, $\overline{ZX}$ is a diameter, and W and Y are points of circle O. How many degrees are in the measure of angle WXY?



ANSWERS

① _____ in.

② _____ cm^2

③ _____ °

CATEGORY 3 - MYSTERY
MARCH, 1997 - MEET #4

① Oliver owns a 360-acre farm. He plants potatoes on 1/3 of the farm, and rutabagas on 2/5 of the remaining acres. Half of the rest of the farm is used for growing lettuce. On how many acres is lettuce grown?

② A painter stood on the middle rung of a ladder. He then went up three rungs, down seven rungs, up two rungs, down five rungs, and up 13 rungs. He then climbed up four more rungs to the top rung, and then stepped up onto the roof of the house. How many rungs did the ladder have?

③ □ is not an even number.
□ is not divisible by 3.
□ is not a prime number.
□ is greater than 50.
□ is not divisible by 11.
□ is less than 70.

What is the value of □?

ANSWERS

① _____

② _____

③ _____

CATEGORY 4 – ARITHMETIC
MARCH, 1997 – MEET #4

① The price tag on the big-screen TV was $975.79. Including sales tax, Homer paid $1044.10. To the nearest whole percent, what was the sales tax rate?

② Items at a store were mistakenly discounted 20%. By what per cent must the discounted prices be marked up in order to be the same as the original prices?

③ Abbot invested $9000 in an institution which paid 18% simple interest per year. Costello invested $9000 in a different institution which paid $1\frac{1}{2}$% per month interest, compounded monthly, where each month's interest was rounded to the nearest cent. Each man invested his money for six months. This is a two-part question: A. Who earned more interest during the six month period, and B. How much more?

ANSWERS

① _____ %

② _____ %

③ A. _____ (who?)
$ _____ (how much more?)

CATEGORY 5 – ALGEBRA
MARCH, – 1997 – MEET #4

① The sum of four consecutive even integers is 548.
If A = the smallest of the four integers, and
C = the largest of the four integers, then
Find the product of A · C.

② Jethro peaked through a hole in the barnyard fence, and saw hens and cows – that's right, just hens and cows! When he counted the legs of all the animals, the total was 378. When he counted the heads of all the animals, the total was 131. Hens are two-legged animals, while each cow has four legs. Assuming that each animal has one, and only one, head, how many cows are in the barnyard?

③ Two birds leave two locations which are 578 miles apart and fly, without stopping, continually back and forth between these two places. One bird flies at 16 miles per hour, and the other at 18 miles per hour. After how many hours will they meet for the second time?

ANSWERS

① _____

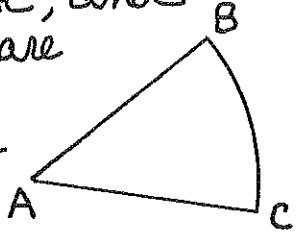
② _____

③ _____ hours

CATEGORY 6 - TEAM QUESTIONS
MARCH, 1997 - MEET #4

① The seven members of the Cycling Club have ridden the following number of miles, respectively: 674; 892; 460; 513; 578; 630; 621. When Tory joins the club next month, she says that the mean number of miles will increase by two miles. Assuming that none of the original seven members will ride until Tory joins, how many miles will Tory have ridden by the time she joins the club?

② In the diagram to the right, $\overline{AB}$ and $\overline{AC}$ are radii of the same circle. The region is enclosed by $\overline{AB}$, $\overline{AC}$, and arc $\overset{\frown}{BC}$. The area of the region is $N \cdot \pi$ square units. Find the value of N, rounded to the nearest whole number, if AB = 8 units, and the measure of angle BAC = 49°.



③ If $\frac{x+72}{x} = \frac{31}{13}$, then find the value of $\sqrt{\frac{1}{2}x - 1}$.

④ Refer to the diagram in problem #2. If the perimeter of the figure is $M \cdot \pi + R$, then find the value of M+R, rounded to the nearest whole number.

⑤ Find the sum of the first 258 terms of the following sequence: 19 25 31 37 43 ...

⑥ If A, B, C, D, and E represent the answers to #1-5 above, respectively, then evaluate:

$$\frac{D\left(\sqrt{(D+C-1)^2 + E}\right)}{B}$$

ANSWERS

① _____ = A

② _____ = B

③ _____ = C

④ _____ = D

⑤ _____ = E

⑥ _____

SOLUTION KEY – March, 1997

CATEGORY 1

① 4428.675
② 20,254
③ 892

① Multiply each number in the sequence by 1.5 to get the next number:

Term #:	1	2	3	4	5
Value:	51.2	76.8	115.2	172.8	259.2

Term #:	6	7	8	9
Value:	388.8	583.2	874.8	1312.2

Term #:	10	11	12
Value:	1968.3	2952.45	4428.675

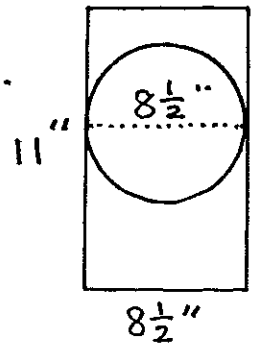
② This problem is equivalent to "Find the sum of the first 82 terms of the following sequence: 4, 10, 16, 22, ..."
The 82nd term is two less than the 82nd multiple of 6: $82(6) - 2 = 490$.
The sum is $\frac{82}{2}(4+490) = 41(494) = 20254$.

③ $880 \div 26$ is $33.846\ldots$. So, the 34th multiple of 26 is between 880 and 900. $(34)(26) = 884$. The 884th letter is Z, as is any multiple of 26. H is the 8th letter of the alphabet. $884 + 8 = 892$.

CATEGORY 2

① 55.994
② 113.04
③ 57

① The largest circle has, at greatest, an $8\frac{1}{2}$" diameter, or $4\frac{1}{4}$" radius.
Area $= \pi r^2 \approx 3.1(4.25^2)$
$\approx 3.1(18.0625) \approx 55.99375$
≈ 55.994 (rounded to the nearest thousandth).



② The diameters of circles A and B are 6 cm and 12 cm, respectively.
Area of $\odot A = \pi r^2 \approx 3.14(3^2) \approx 3.14(9) \approx 28.26$
Area of $\odot B = \pi r^2 \approx 3.14(6^2) \approx 3.14(36) \approx 113.04$

(continued on next page)

Solution Key - continued

Area of $\odot C = \pi r^2 \approx 3.14(9^2) \approx 3.14(81) \approx 254.34$
Area of shaded region $\approx$ Area of $\odot C - (\odot A + \odot B)$
$\approx 254.34 - (28.26 + 113.04)$
$\approx 254.34 - 141.3$
$\approx 113.04 \text{ cm}^2$

③ $\angle W$ and $\angle Y$ are both right angles, since each is inscribed in a semi-circle. Using the theorem that the sum of the angles of any triangle is 180°, $m\angle WXZ = 39°$, $m\angle ZXY = 18°$, and their sum ($m\angle WXY$) $= 39 + 18 = 57°$.

CATEGORY 3

① 72
② 21
③ 65

① Potatoes : $\frac{1}{3}$ of 360 = 120 acres.
Rutabagas : $\frac{2}{5}$ of (360 − 120) = $\frac{2}{5}$ of 240 = 96 acres
Lettuce : $\frac{1}{2}$ of the remaining acres
$= \frac{1}{2}[360 - (120 + 96)]$
$= \frac{1}{2}[360 - 216]$
$= \frac{1}{2}[144]$
$= 72$ acres

② A ladder with a middle rung has an odd number of rungs, with as many rungs below the middle as there are above it. There are 10 rungs above the middle one, so 10 are below, plus the middle rung, make a total of 21.

③ By the process of elimination, $\square = 65$.

SOLUTION KEY – continued ...

CATEGORY 4

① 7 (%)

② 25 (%)

③ Costello (#) 30.99

(Both parts must be answered correctly for the student to earn two points.)

① SALES TAX RATE = $\frac{\text{amount of sales tax}}{\text{cost of item}} = \frac{1044.10 - 975.79}{1044.10}$

$= \frac{68.31}{1044.10} \approx 0.0654\ldots \approx 7\%$

② This problem could be solved (A) algebraically, or (B) by choosing a convenient number, like #100:

(A) Let x = original price
n = % of markup

$$(x - .2x) + n(x - .2x) = x$$
$$.8x + n(x - .2x) = x$$
$$.8x + n(.8x) = x$$
$$.8x(1 + n) = x$$
$$.8(1 + n) = 1$$
$$1 + n = \frac{1}{.8}$$
$$1 + n = 1.25$$
$$n = .25$$
$$n = 25\%$$

(B) #100 − 20% of #100 = #80

#80 + □ = #100

□ = #20

$\frac{\#20}{\#80} = .25 = 25\%$

③ Abbot's interest: $I = prt$

$= (9000)(.18)(.5)$, where 6 months = .5 year

$= \#810$

Costello's interest:

1st month: $(9000)(.015) = \#135$

2nd month: $(9135)(.015) = \#137.025 \approx \#137.03$

3rd month: $(9272.03)(.015) \approx \#139.080 \approx \#139.08$

4th month: $(9411.11)(.015) \approx \#141.166 \approx \#141.17$

(next page)

5^{th} month: $(9552.28)(.015) \approx \#143.284$
$\approx \#143.28$

6^{th} month: $(9695.56)(.015) \approx \#145.433$
$\approx \#145.43$

$\therefore$ Castello's interest $= 135 + 137.03 + 139.08 + 141.17 + 143.28 + 145.43$
$= \#840.99$

$\therefore$ Castello earned more interest. How much more?
$\#840.99 - 810 = \#30.99$

CATEGORY 5

① 18,760
② 58
③ 51

① Let x = 1st integer
$x+2$, $x+4$, $x+6$ } = next three even integers

$x + (x+2) + (x+4) + (x+6) = 548$
$4x + 12 = 548$
$4x = 536$
$x = 134$ (smallest) $= A$
$x+6 = 140$ (largest) $= C$

$\therefore A \cdot C = (134)\cdot(140)$
$= 18,760$

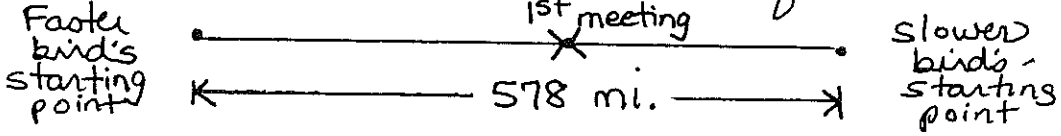
② Let H = # of hens
C = # of cows

1. $C + H = 131$ (each animal has one head.)
2. $4C + 2H = 378$ (each cow has 4 legs, each hen has 2 legs.)

1. $2C + 2H = 262$ (mult equ. 1. by 2)
2. $4C + 2H = 378$

$2C = 116$ (subtract equ. 1. from equ. 2.)
$C = 58$ Done! The question asks, "How many cows ..."

③ When the two birds meet for the first time, they have flown for a combined total of 578 miles:



③ continued...
After meeting each other for the first time, each bird continues to fly toward each others' starting point, reverses direction, then meet each other a second time. The distance covered between their first meeting and second meeting is 578(2), or 1156 miles. So, the total distance covered, from start to finish, is 578 + 1156, or 1734 miles.
Let x = # of hours each bird flies
$16x$ = distance covered by slower bird
$18x$ = distance covered by faster bird

$16x + 18x = 1734$
$34x = 1734$
$x = 51$
∴ They meet for the second time after flying $\underline{51}$ hours.

CATEGORY 6

① 640
② 9
③ 5
④ 18
⑤ 203,820
⑥ 904

① The mean (average) of the seven original members is $(674 + 892 + 460 + 513 + 578 + 630 + 621) \div 7 = 624$. After Tory joins, the mean will increase to 626, which is an increase of two miles. ∴ Tory must ride 16 miles more than the average to bring the average of an 8-member club to 626. So Tory will have ridden 624 + 16, or 640 miles.

② The area of the wedge $= \frac{49}{360}\pi r^2$, or $\frac{49}{360} \cdot \pi \cdot 8^2$, $\approx 8.71\pi$. ∴ $N = 9$, which is 8.71 rounded to the nearest whole number.

③ Using the concept that if two ratios (or fractions) are equal, then their cross-products are equal:

$$\frac{x+72}{x} = \frac{31}{13}$$

$13(x+72) = 31x$
$13x + 13(72) = 31x$
$13x + 936 = 31x$
$936 = 18x$
$52 = x$

Now evaluate:

$$\sqrt{\tfrac{1}{2}x - 1}$$
$$= \sqrt{\tfrac{1}{2}(52) - 1}$$
$$= \sqrt{26 - 1}$$
$$= \sqrt{25}$$
$$= 5$$

Category 6, continued ...

④ Perimeter = arc + 2 · radius

$$= \frac{49}{360} \cdot 2\pi r + 2(8)$$

$$= \frac{49}{360}(16\pi) + 16$$

$$\approx 2.1\overline{7}\,\pi + 16$$

Rounded to the nearest whole number, M+R = 2+16 = 18.

⑤ The sequence can be compared to the multiples of 6, where each term is 13 more than the multiple of 6 which corresponds to the number of the term. The 258th term is 6(258)+13, or 1561. The sum of the first 258 terms is 258(19+1561) ÷ 2, or 203,820.

⑥ $$\frac{D\left(\sqrt{(D+C-1)^2 + E}\right)}{B}$$

$$= \frac{18\left(\sqrt{(18+5-1)^2 + 203,820}\right)}{9}$$

$$= \frac{18\left(\sqrt{(22)^2 + 203,820}\right)}{9}$$

$$= \frac{18\left(\sqrt{484 + 203,820}\right)}{9}$$

$$= \frac{18\left(\sqrt{204,304}\right)}{9}$$

$$= \frac{18(452)}{9} \rightarrow = 904$$

$$= \frac{8136}{9}$$