INTERMEDIATE MATH LEAGUE

January 23, 1997
Division 4

MEET #3

MCCALL MIDDLE SCHOOL

CATEGORY 1 - NUMBER THEORY

JANUARY, 1997 - MEET #3

1. Express the base 3 numeral 10110 as a base 10 numeral:

$$10110_{\text{Base }3} = \underline{\qquad}_{\text{Base }10}$$

2. Barney sent \$1,295 in U.S. money to his friend, Fred. However, Fred lives on the island of Octos, where the money system is based on the number eight and its powers (base 8), and needs to convert the U.S. bills into Octos bills, getting as few bills as possible. How many 64-bills will he get?

3. In the 1992 presidential election campaign, Bob Dole received some write-in votes. Bill Clinton got 5000 times as many votes with 67,800,000. How many votes did Bob Dole receive? Express your answer in scientific notation.

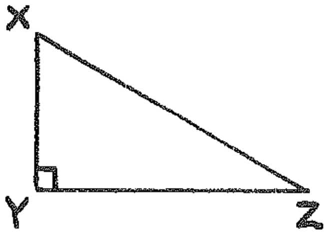
ANSWERS

1. ________
2. ________
3. ________

CATEGORY 2 - GEOMETRY

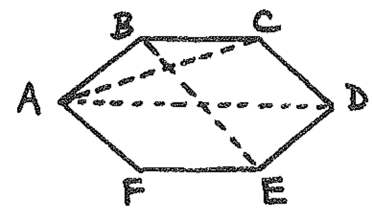
JANUARY, 1997 - MEET #3

① An exterior angle of a regular polygon measures $24°$. How many sides does the polygon have?

② 

Slim and Jim both start at point X and race to point Z. $XY = 18$ meters and $XZ = 30$ meters. Both runners race at the same rate. Both runners try to get to point Z as quickly as possible, except that Slim must first pass through point Y. Slim runs how many more meters than Jim?

③ A diagonal of a polygon is a segment which connects any two non-consecutive vertices. For example, referring to the diagram below, $\overline{AC}$ and $\overline{BE}$ + $\overline{AD}$ are diagonals, but $\overline{AB}$ and $\overline{AF}$ are not. What is the total number of diagonals which can be drawn for a 15-sided regular polygon?



ANSWERS

① ______

② ______

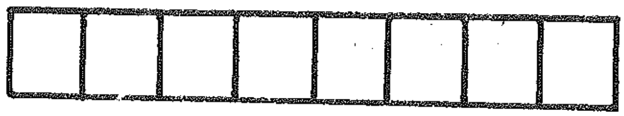
③ ______

CATEGORY 3 - MYSTERY

JANUARY, 1997 - MEET #3

① If you include 17 and 83, how many whole numbers are there from 17 to 83 ?

② What is the total number of rectangles which can be traced in the figure below, formed by placing eight squares in a row :



③ The sum of two whole numbers is 28, and their product is 132. What is the larger of the two numbers ?

ANSWERS

① ______

② ______

③ ______

CATEGORY 4 – ARITHMETIC
JANUARY, 1997 – MEET #3

1. Evaluate: $(4^3)(2^{-3})(6^0)(5^1)$

2. Find the value of  if

 is a perfect square,
 is a perfect cube,
 > 1,
and  < 100.

3. Evaluate:

$$\left(\sqrt{\left(\sqrt[6]{\left(\sqrt{\sqrt{16^2}}\right)^3}\right)^4}\right)^5$$

ANSWERS

1. ______
2. ______
3. ______

CATEGORY 5 – ALGEBRA

JANUARY, 1997 – MEET #3

① Evaluate: $|8-2| + |2-8|$

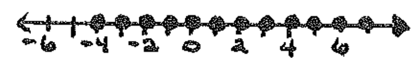
② Find the sum of all solutions to the following equation:

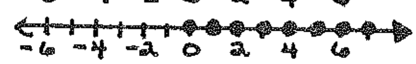
$$|4x-2| = 54$$

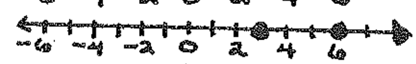
③ Depending on the domain (replacement numbers for the variable), the graph of an inequality may be represented in various ways. Among the graphs drawn below are many which represent the solution set for $x > -4$, given different domains. Your job is to match each domain for $x > -4$ with the appropriate graph, then write the letter of that graph into box corresponding to the number of the problem. #2 has been done for you, as graph U represents all solutions to $x > -4$ which are non negative real numbers. Note: Some graphs will not be used.

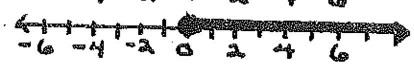
Domain

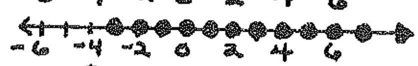
1. Whole numbers
2. Non-negative real numbers
3. Natural numbers
4. Integers
5. Counting numbers
6. Positive multiples of 3
7. Non-negative integers
8. Real numbers

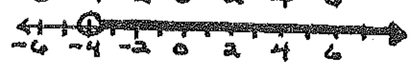
N 

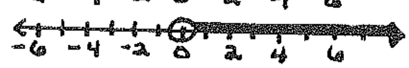
S 

I 

U 

P 

E 

M 

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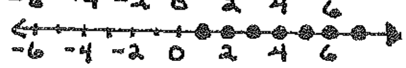
R 

ANSWERS

① ______

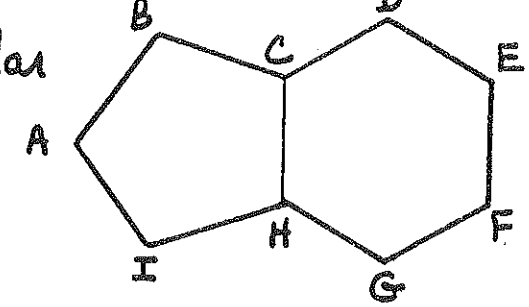
② ______

③

1	2	3	4	5	6	7	8
	U						

CATEGORY 6 - TEAM QUESTIONS
JANUARY, 1997 - MEET #3

① If half of a number, 25% of the number, and 0.1 times the number have a sum of 102, then what is that number?



② Regular pentagon ABCHI and regular hexagon CDEFGH share a common side, $\overline{CH}$. How many degrees are in exterior angle BCD?

③ When the fractional equivalent of $0.0\overline{36}$ is in lowest terms, what is the numerator?

④ What is the quotient of the sum of the odd prime factors of 7854 and the sum of its even prime factors?

⑤ What is the greatest possible difference, in square centimeters, between the areas of two rectangles if the perimeter of each rectangle is 144 cm, and all lengths and widths are natural numbers?

⑥ If A, B, C, D, and E represent the answers to #1-5 above, respectively, then evaluate:

$$\frac{A+B}{\left(\sqrt{\sqrt{E}-D}\right)^{C}}$$

Express your answer as a decimal.

ANSWERS

① ________ = A

② ________ = B

③ ________ = C

④ ________ = D

⑤ ________ = E

⑥ ________

SOLUTION KEY – JANUARY, 1997

Numb Th

CATEGORY 1

① 93

② 4

③ 1.356×10^4

① $\underline{1}\ \underline{0}\ \underline{1}\ \underline{1}\ \underline{0}$

81 27 9 3 1 ← place values

$81 + 9 + 3 = 93$

② Base 8 place values needed : 1, 8, 64, 512

$$\begin{array}{r} 2 \\ 512\overline{)1295} \\ \underline{1024} \\ 271 \end{array} \qquad \begin{array}{r} 4 \\ 64\overline{)271} \\ \underline{256} \\ 15 \end{array}$$

← We can stop here and say that four 64-bills is what Fred will get.

③ $\dfrac{67,800,000}{5000} = 13560$

$= 1.356 \times 10^4$

CATEGORY 2

① 15

② 12

③ 90

① The sum of the exterior angles of any convex polygon, which includes regular polygons, is 360°. ∴ $360 \div 24 = 15$ angles. A polygon of 15 angles has 15 sides.

② First, use the Pythagorean Theorem to find YZ :

$$\begin{aligned} (XY)^2 + (YZ)^2 &= (XZ)^2 \\ 18^2 + (YZ)^2 &= 30^2 \\ 324 + (YZ)^2 &= 900 \\ (YZ)^2 &= 576 \\ YZ &= \sqrt{576} \\ YZ &= 24 \end{aligned}$$

→ Jim runs 30 m. Slim runs 18+24, or 42 m. ∴ Slim runs 12 m longer than Jim. $(42 - 30 = 12)$

③ A student may wish to draw the 15-gon and actually count all the diagonals, or a pattern may be discovered for the number of diagonals by looking at polygons of 3, 4, 5, ... sides :

# of sides	# of diagonals	
3	0	+2
4	2	+3
5	5	+4
6	9	+5
7	14	
⋮	⋮	
n	$\dfrac{n(n-3)}{2}$	

Adding a side also adds one more diagonal than was previously added. An accurate answer may be reached quickly this way. Or else a generalization may be reached :

$$\frac{15(15-3)}{2} = \frac{15(12)}{2} = 90.$$

Solution Key - continued ...

Myst

CATEGORY 3

① 67

② 36

③ 22

① The numbers of whole numbers from A to B, inclusive, where $A < B$, is $B - A + 1$. $83 - 17 + 1 = 67$.

②

dimensions of rectangle	#
1x1	8
1x2	7
1x3	6
⋮	⋮
1x8	1

The solution is equivalent to finding the sum $1+2+3+\ldots+8 = 36$.

③ Although this looks like an algebra problem, it can be easily solved without algebra. Try picking an arbitrary pair of numbers whose sum is 28, find the product, then adjust accordingly:

#	other #	sum	product	
10	18	28	180	(too large)
9	19	28	171	(still too large)
8	20	28	160	(again, too large)
7	21	28	147	(this is getting ridiculous!)
6	22	28	132	(ah - just right!)

∴ The larger number is 22.

CATEGORY 4

① 40

② 64

③ 1024

① $(4^3)(2^{-3})(6^0)(5^1) = (64)(\frac{1}{8})(1)(5) = 40$

② Since  < 100, the only cubes which are less than 100 are 1, 8, 27, and 64. The only square which is greater than 1 is 64.

③ In sequence, starting with the innermost operation: $\sqrt{16^2} = 16$, $\sqrt{16} = 4$, $4^3 = 64$, $\sqrt[6]{64} = 2$, $2^4 = 16$, $\sqrt{16} = 4$, $4^5 = 1024$.

CATEGORY 5

① 12

② 1

③ SURPRISE

① $|8-2| + |2-8| = |6| + |-6| = 6+6 = 12$

② $4x - 2 = 54$ or $4x - 2 = -54$
$4x = 56$ or $4x = -52$
$x = 14$ or $x = -13$

The sum is $14 + (-13) = 1$

③ Surprise!

Solution Key – continued ... JAN 1997

CATEGORY 6

① 120
② 132
③ 2
④ 19
⑤ 1225
⑥ 15.75

① $.5x + .25x + .1x = 102$
$.85x = 102$ → $x = 102 \div .85$
$x = 120$

② Extend $\overline{HC}$ to a point X. $\angle BCX$ is an exterior angle of the pentagon, and measures $360 \div 5 = 72°$. $\angle XCD$ is an exterior angle of the hexagon, and measures $360 \div 6 = 60°$. Exterior angle BCD $= \angle BCX + \angle XCD = 72 + 60 = 132°$.

③
$$\begin{aligned} 100x &= 3.6\overline{36} \\ \text{Let } x &= 0.0\overline{36} \\ \hline 99x &= 3.6 \end{aligned}$$
→ $x = \frac{3.6}{99} = \frac{36}{990} = \frac{18}{495} = \frac{2}{55}$

∴ The numerator is 2.

④
$$\begin{aligned} 7854 &= 2 \cdot 3927 \\ &= 2 \cdot 3 \cdot 1309 \\ &= 2 \cdot 3 \cdot 7 \cdot 187 \\ &= 2 \cdot 3 \cdot 7 \cdot 11 \cdot 17 \end{aligned}$$

Sum of odd factors
$= 3 + 7 + 11 + 17$
$= 38$

Sum of even factors $= 2$

$\therefore 38 \div 2 = 19$.

⑤ A rectangle of fixed perimeter has its maximum area when it is a square ($144 \div 4 = 36$, which is the length of one side, so its area is $36 \times 36 = 1296$), and has its minimum area when its shape is "long" – in this case, a 1×71, whose area is 71.
∴ The greatest possible difference is $1296 - 71 = 1225$.

⑥
$$\frac{A+B}{\left(\sqrt{\sqrt{E} - D}\right)^C}$$

$$= \frac{120 + 132}{\left(\sqrt{\sqrt{1225} - 19}\right)^2}$$

$$= \frac{252}{\left(\sqrt{35 - 19}\right)^2}$$

$$= \frac{252}{\left(\sqrt{16}\right)^2}$$

$$= \frac{252}{16}$$

$$= 15.75$$