CATEGORY 1 - MYSTERY

NOVEMBER, 1996 - MEET #2

① A spider was walking on a number line which was on a poster above the blackboard. It was walking from the negative numbers toward the positive numbers, starting at -7. He saw a fly on the number $+23$ and rushed toward it, but was "rescued" by the teacher at a point which was $2/5$ of the way from -7 to $+23$. On which number was the spider "rescued"?

② A machine can fill 160 boxes of candy in 8 minutes. Each box has 100 pieces of candy in it. How many boxes can be filled in one hour?

③ What is the unit's digit (the "ones" place) of 3^{51}?

ANSWERS

① ______

② ______

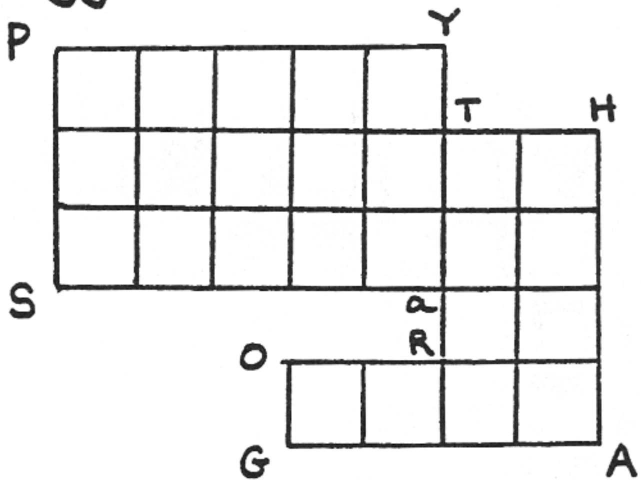
③ ______

CATEGORY 2 - GEOMETRY

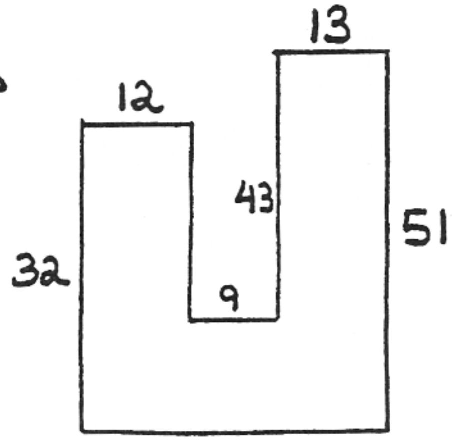
NOVEMBER, 1996 - MEET #2

① The perimeter of a regular octagon is 72 cm. How many square centimeters are in the area of a square, where each side of the square is 3 cm longer than a side of the octagon?

② Squares have been arranged to form the shape below. The area of polygon PYTHAGORAS is 400 square units. What is the number of units in the perimeter of polygon PYTHAGORAS?



③ Find the numbers of units in the perimeter of the figure below. All angles are right angles...



ANSWERS

① ______ sq. cm

② ______ units

③ ______ units

CATEGORY 3 - NUMBER THEORY
NOVEMBER, 1996 - MEET #2

① @ = the sum of all composite numbers which are greater than 80 but less than 90.

\# = the sum of all prime numbers which are greater than 50 but less than 60.

Find the value of @ − #.

② What is the smallest prime number which is greater than 140?

③ GCF(x, y) means "the greatest common factor of x and y". LCM(x, y) means "the lowest common multiple of x and y".

GCF$(x, y) = 6$
LCM$(x, y) = 72$
$x = 18$

Find the value of y.

ANSWERS

① ______

② ______

③ ______

CATEGORY 4 - ARITHMETIC

NOVEMBER, 1996 - MEET #2

① An operation involving two numbers, N and P, is defined as follows:

$$N \odot P = \frac{\frac{1}{N} - \frac{1}{P}}{\frac{1}{N} + \frac{1}{P}}.$$

Find the value of $8 \odot 12$. Express your answer as a fraction in lowest terms.

② Find the positive difference between 0.8 and $0.\overline{8}$. Express your answers as a fraction in lowest terms.

③ A typical school day starts at 8:00 A.M. and ends at 2:46 P.M. Included are seven class periods, six of which are 48 minutes long, and one is an hour in length. There is a half-hour lunch period. Four minutes passing time is allowed between periods. To the nearest whole percent, what percent of a typical school day is not spent in class?

ANSWERS

① ______

② ______

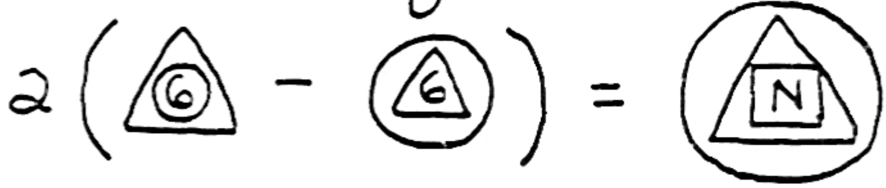
③ ______ %

CATEGORY 5 -- ALGEBRA

NOVEMBER, 1996 - MEET #2

1. Three consecutive odd numbers have a sum of 261. What is the largest of those three numbers?

2. If  means "subtract 2 from E",
and  means "multiply E by 5,"
and  means "square E",
then find the value of N, such that



and N is a positive number. Express your answer as a decimal.

3. The total number of spheres needed to create a triangular pyramid is given by the formula

$$N = \frac{x^3}{6} + \frac{x^2}{2} + \frac{x}{3}$$

where x is the number of layers in the pyramid, and N is the total number of spheres. How many spheres are in the 8th layer?

ANSWERS

1. ______

2. N: ______

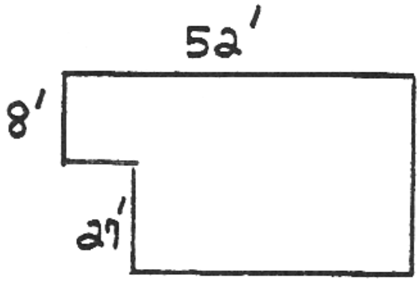
3. ______

CATEGORY 6 - TEAM QUESTIONS

NOVEMBER, 1996 - MEET #2

① Find the sum of all prime numbers between 180 and 200.

② A cement path, which is five feet wide, surrounds a swimming pool like the one pictured to the right. All angles are right angles. Find the number of square feet in the area of the cement path.



③ Find the smallest positive integer, N, such that the product $120 \cdot N$ is a perfect square.

④ The average person can speak 120 words in $1\frac{1}{2}$ minutes, and can write 96 words in $2\frac{2}{3}$ minutes. In a ten-minute period, how many more words can be spoken than written by the average person?

⑤ Bill and Bob jog around an oval track which has a circumference of 196 meters. Bill's rate is 212 meters per minute, and Bob's is 184 meters per minute. In how many minutes will Bill be one full lap ahead of Bob, if they start at the same place at the same time?

⑥ If A, B, C, D, and E represent the answers to #1-5 above, then evaluate:

$$\left(\frac{\frac{B+D}{C}+\sqrt{A}}{C+E+2}\right)^3$$

ANSWERS

① ________ = A

② ________ = B

③ ________ = C

④ ________ = D

⑤ ________ = E

⑥ ________

SOLUTION KEY - NOVEMBER, 1996

Number Theory

CATEGORY 3

① 481

② 149

③ 24

① $⑥ = 81 + 82 + 84 + 85 + 86 + 87 + 88 = 593$

$\# = 53 + 59 = 112$

$\therefore ⑥ - \# = 593 - 112 = 481$

Note : 51, 57, 81, and 87 are composite, as they are divisible by 3.

② 141 and 147 are multiples of 3, and $143 = 11 \times 13$. 145 is divisible by 5. All of the even numbers are divisible by 2.

③ Using the idea that the product of two numbers is the same as the product of their GCF and LCM :

$$x \cdot y = GCF(x, y) \cdot LCM(x, y)$$
$$18 \cdot y = 6 \cdot 72$$
$$18 \cdot y = 432$$
$$y = 432 \div 18$$
$$y = 24$$

Geometry

CATEGORY 2

① 144

② 112

③ 218

① One side of the octagon is $72 \div 8 = 9$ cm. One side of the square is $9 + 3 = 12$ cm. The area of the square is $12 \times 12 = 144$ sq. cm.

② Area of one square is $400 \div 25 = 16$ sq. units. One side of a square is $\sqrt{16} = 4$ units. $\therefore$ Perimeter of PYTHAGORAS $= 28 \cdot 4 = 112$ units.

③ There are two lengths with no labelled measure. The horizontal base is as long as the sum of the three horizontal lengths which are given : $12 + 9 + 13 = 34$. The vertical unknown length can be found as follows : The right "tower" is 19 taller than the left one $(51 - 32 = 19)$. $\therefore 43 - 19 = 24$. The perimeter is the sum of all lengths :

$$34 + 24 + 32 + 12 + 9 + 43 + 13 + 51 = 218.$$

Solution Key - continued ...

November 1996

Mystery CATEGORY 1	
① 5 ② 1200 ③ 7	① $^{+}23$ is 30 units to the right of -7. $\frac{2}{5}$ of 30 is 12, so $-7+12=5$. The 12 was added to -7, because the spider was moving from left to right. ② The "100 pieces of candy" is irrelevant data. $160 \div 8 = 20$ boxes per minute. 60 min. = 1 hr., so $\frac{20 \text{ boxes}}{1 \cancel{\text{ minute}}} \cdot \frac{60 \cancel{\text{ minutes}}}{1 \text{ hr.}} = 1200$ boxes/hr. ③ There is a cyclical pattern in consecutive powers of 3: $3^0 = 1$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, $3^6 = 729$, $3^7 = 2187$, etc. The units digits are 1, 3, 9, 7, 1, 3, 9, 7, etc., repeating in groups of four. In 3^{51}, 51 is one less than a multiple of four. Powers of 3 whose exponent is one less than a multiple of 4 always have a units digit of 7.

Arithmetic CATEGORY 4	
① $\frac{1}{5}$ ② $\frac{4}{45}$ ③ 14 (%)	① $\dfrac{\frac{1}{N}-\frac{1}{P}}{\frac{1}{N}+\frac{1}{P}} = \dfrac{\frac{1}{8}-\frac{1}{12}}{\frac{1}{8}+\frac{1}{12}} = \dfrac{\frac{3}{24}-\frac{2}{24}}{\frac{3}{24}+\frac{2}{24}} = \dfrac{\frac{1}{24}}{\frac{5}{24}} = \frac{1}{\cancel{24}} \cdot \frac{\cancel{24}}{5} = \frac{1}{5}$ ② $0.\overline{8} - 0.8 = \frac{8}{9} - \frac{8}{10} = \frac{80}{90} - \frac{72}{90} = \frac{8}{90} = \frac{4}{45}$ ③ The time not spent in class can be easily figured by subtracting the amount of class time ($6 \cdot 48 + 60 = 348$ minutes) from the # of minutes in the entire school day (2:46 P.M. − 8:00 A.M. = 6 hr. 46 min., or 406 minutes): $406 - 348 = 58$ minutes. Alternative: Add the # of minutes of passing time ($7 \times 4 = 28$) to the # of minutes of the lunch period (30): $28 + 30 = 58$. (Seven passes are necessary because their are seven classes and one lunch period.) $\frac{\text{\# of min. not in class}}{\text{Total \# min in school day}} = \frac{58}{406} \approx 0.142 \approx 14\%$

Solution Key - continued ... November 1996

CATEGORY 5

① 89

② 1.6

③ 36

① $\left.\begin{array}{l} x \\ x+2 \\ x+4 \end{array}\right\}$ three consecutive odd numbers

$$x + x+2 + x+4 = 261$$
$$3x + 6 = 261$$
$$3x = 255$$
$$x = 85$$
$$\therefore x+4 = 89 \text{ (the largest!)}$$

Or, you could reason that their median must be 261/3 or 87, so the numbers are 85,87,89.

② Using more standard operation symbols, the equation can be written

$$2\left((6^2-2)-(6-2)^2\right) = (5N-2)^2$$
$$2[(36-2)-(6-2)^2] = (5N-2)^2$$
$$2[(34)-(16)] = (5N-2)^2$$
$$2[18] = (5N-2)^2$$
$$36 = (5N-2)^2$$
$$6 = 5N-2 \quad \text{or} \quad -6 = 5N-2$$
$$8 = 5N \qquad -4 = 5N$$
$$\frac{8}{5} = N \qquad -.8 = N \text{ (extraneous)}$$
$$1.6 = N$$

(the question asks for a decimal which is positive.)

③ The # of spheres in the 8th layer can be found by subtracting the total # of spheres in a 7-layer configuration from the total # of spheres in an 8-layer configuration:

For 8 layers, $N = \frac{8^3}{6} + \frac{8^2}{2} + \frac{8}{3}$

$= \frac{512}{6} + \frac{64}{2} + \frac{8}{3}$

$= 85\frac{1}{3} + 32 + 2\frac{2}{3} = 120$

Or, factor the x first:
x(1/3+(x/2)(1+x/3))

For 7 layers, $N = \frac{7^3}{6} + \frac{7^2}{2} + \frac{7}{3}$

$= \frac{343}{6} + \frac{49}{2} + 2\frac{1}{3}$

$= 57\frac{1}{6} + 24\frac{1}{2} + 2\frac{1}{3} = 84$

$\therefore$ 120 − 84 = 36 spheres in the 8th layer.

Team Round

CATEGORY 6

① 961

② 970

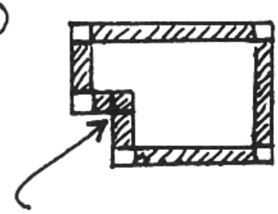
③ 30

④ 440

⑤ 7

⑥ 8

① $181+191+193+197+199 = 961$ (Note: 183 and 189 are multiples of 3, and 187 is divisible by 11.)

② 

The shaded rectangles are the portions of the cement walk which "protrude" from the sides of the pool, $= 5\cdot(52+52+8+27+8+27)$
$= 5\cdot(174) = 870$. However, this square is counted twice in doing so,
$\therefore\ 870-25 = 845$. Now add in the areas of the five corner regions $(5\cdot 25 = 125)$,
$\therefore\ 845+125 = 970$. Many other techniques possible.

③ $120 = 2^3\cdot 3\cdot 5$. For $120\cdot N$ to be a perfect square, $N = 2\cdot 3\cdot 5$, or 30. Then $120\cdot 30 = 3600$, which is 60^2.

④ Speaking: $120 \div 1\frac{1}{2} = 80$ words per minute, or 800 words in 10 minutes.
Writing: $96 \div 2\frac{2}{3} = 36$ words per minute, or 360 words in 10 minutes.

$800 - 360 = 440$.

⑤ Let x = # of minutes each boy jogs.
Bill's distance : $212x$
Bob's distance : $184x$
Bill's distance must be 196 meters longer than Bob's (one lap = 196 meters), so

$$212x - 184x = 196$$
$$28x = 196$$
$$x = 7 \text{ (minutes)}$$

⑥ $$\left(\frac{\frac{B+D}{C}+\sqrt{A}}{C+E+2}\right)^3$$

$$= \left(\frac{\frac{970+440}{30}+\sqrt{961}}{30+7+2}\right)^3$$

$$= \left(\frac{\frac{1410}{30}+31}{39}\right)^3$$

$$= \left(\frac{47+31}{39}\right)^3$$

$$= \left(\frac{78}{39}\right)^3$$

$$= (2)^3$$

$$= 8$$