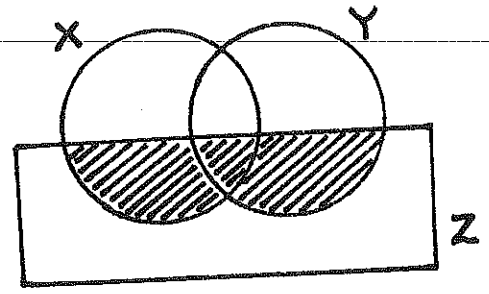
CATEGORY 1 - Number Theory
March, 1996 - Meet #5

① Which of the following sets (A, B, C, D, or E) is represented by the Venn diagram to the right?

(shaded portion only)

$A = X \cup (Y \cap Z)$
$B = (X \cup Y) \cup Z$
$C = (X \cap Y) \cap Z$
$D = (X \cup Y) \cap Z$
$E = (X \cap Y) \cup Z$



② If $N = \{$ positive multiples of $6\}$, and
$M = \{$ positive factors of $72\}$, and
$P = \{$ all two-digit numbers where the unit's digit is larger than the ten's digit $\}$.
then find the sum of all elements of set W, if $W = (N \cap M) \cap P$.

③ Of the 683 students at Al Capp Middle School,
248 wear hats,
412 wear sneakers,
364 are girls,
53 are girls who wear hats,
119 are girls who wear sneakers, and
35 are girls who wear hats and sneakers.
How many boys, who are students at Al Capp Middle School, wear sneakers, but not hats? Every boy wears either a hat or sneakers or both.

ANSWERS

① _____

② _____

③ _____

CATEGORY 2 - Geometry
March, 1996 - Meet #5

① If the rainfall on Tuesday was $2\frac{1}{4}$ inches, then find the number of cubic inches of rain that fell onto a rectangular piece of ground which measures 25 feet wide and 100 feet long.

② A golf ball 1.4 inches in diameter has 234 dimples, which are hemispherical indentations, each with a diameter of 0.18 of an inch. What is the surface area of the golf ball in square inches, rounded to the nearest tenth of a square inch? Use $\pi \approx 3.14$.

③ For an irrigation project, a cylindrical tunnel was drilled through a mountain. The diameter of the tunnel was nine feet and its length was 1100 feet. How many cubic yards of material had to be removed? Use $\pi \approx 3.14$.

ANSWERS

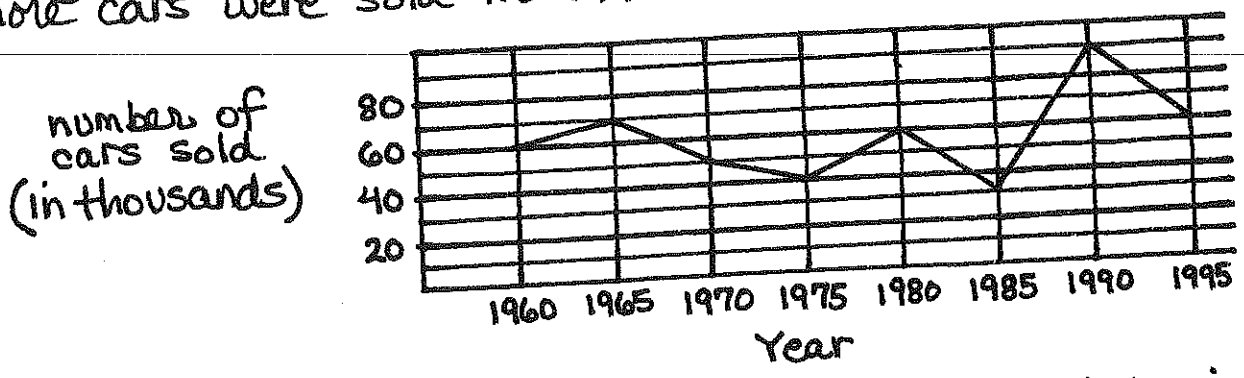
① _____ cu. in.

② _____ sq. in.

③ _____ cu. yd.

CATEGORY 3 - Mystery
March, 1996 - Meet #5

① Using the information in the line graph below, how many more cars were sold in 1990 than in 1980?



② The weekday rate for a phone call between two cities is $0.59 for the first minute and $0.43 for each additional minute. The evening rate between the same two cities is $0.34 for the first minute and $0.21 for each additional minute. How many dollars are saved by making a 38-minute phone call during the evening?

③ Five times seven and seven times three
Add to my age and it will be
As far above six nines and four
As twice my years exceeds a score.
How many years old am I?
(Note: A score = 20)

ANSWERS

① _____ cars

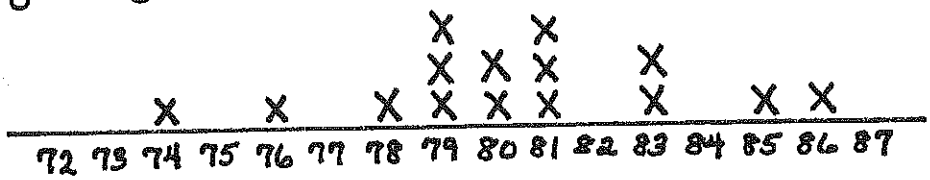
② $_____

③ _____ years

CATEGORY 4 - Arithmetic
March, 1996 - Meet #5

① The probability that Betty will go out with Archie is 40%, while the probability that Veronica will go out with Archie is 25%. What is the probability that both Betty and Veronica will go out with Archie? Express your answer as a common (simplified) fraction.

② Recent test scores for Ms. Euclid's class are displayed in the frequency distribution chart below:



Two students were absent on the day the test was given. When those two students take the test, what must the average of those two scores be so that the class average is 82?

③ A box contains only six dark chocolates, seven milk chocolates, and two white chocolates. If he selects two chocolates from the box, at random (without looking!), what is the probability that they are both dark chocolates? Express your answer as a common fraction.

ANSWERS

① _____

② _____

③ _____

CATEGORY 5 - Algebra
MARCH, 1996 - Meet #5

① There are two values of N which make the equation $N^2 + 11N = 42$ a true statement. What are those two values of N?

② An object which is thrown, hit, or otherwise thrust vertically into the air will follow a path algebraically described by $d = rt - 5t^2$, where d is the height reached after t seconds, having been released at an initial velocity (speed) of r.
Mo threw a ball straight up into air with an initial upward velocity of 65 feet per second, and caught it when it came back down. What is the greatest height, in feet, that the ball reached (above the ground) if Mo released it from five feet above the ground, and caught it also at five feet above the ground?

③ José, who is always trying to outdo Mo, threw his ball straight up from a height of 6 feet above the ground at an initial upward velocity of 74.6 feet per second. When the ball was on its way down, José got distracted, so he did not catch the ball. It landed beside him, 15 seconds after he threw it. To the nearest tenth of a foot, what is the maximum (highest) height above the ground that José's ball reached?

ANSWERS

① ____ and ____

② _____ feet

③ _____ feet

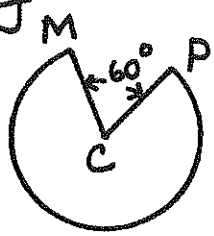
CATEGORY 6 - Team Questions
March, 1996 - Meet #5

① W contains 75 the same number of times that 1509.6 contains $\frac{3}{4}$ of 80% of 340. Find the value of W.

② A cyclist rode from Rockport to Gloucester in 15 minutes with the wind at her back, and returned to Rockport in 20 minutes along the same route with the wind at her face. The entire trip (round trip) was 8 Km. Find the speed of the cyclist, in Km per hour, if the wind did not blow at all.

③ Movie ticket prices are: \$4.25 for children, and \$7.50 for adults. If 329 attended the movie, and paid a total of \$1859.75, then how many were adults?

④ Find the perimeters of the figure to the right, if C is a center. Let $\pi \approx 3.1$. CM and CP are radii. CM = 18.



⑤ One leg of a right triangle is seven inches less than twice as long as the shorter leg. The hypotenuse is fourteen inches longer than the longer leg. How many square inches are in the area of the triangle?

⑥ Using A, B, C, D, E as the answers to questions #1-5, evaluate the following expression:

$$\frac{C + D + 5}{\sqrt[3]{\frac{E}{B}} + \sqrt{\frac{A}{15}} - 1}$$

ANSWERS

① _____ = A
② _____ = B
③ _____ = C
④ _____ = D
⑤ _____ = E
⑥ _____

SOLUTION KEY - MARCH, 1996

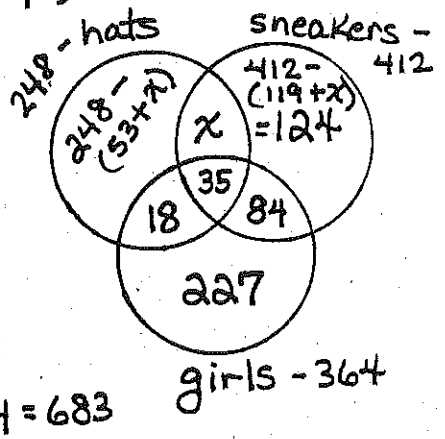
CATEGORY 1

① D
② 90
③ 124

① The diagram shows a shaded region which is where the union of the two circles intersects the rectangle. Hence, answer "D".

② $N = \{6, 12, 18, 24, \ldots\}$
$M = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$
$\therefore N \cap M = \{6, 12, 18, 24, 36, 72\}$
$\therefore (N \cap M) \cap P = \{12, 18, 24, 36\}$
The sum of the elements of W = 12 + 18 + 24 + 36 = 90.

③ Since the first three sets overlap, make an appropriate Venn diagram:



To find the value of x, write an equation which says that the sum of all the parts is 683:

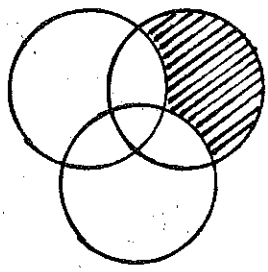
$x + 248 - (53 + x) + 412 - (119 + x) + 18 + 35 + 84 + 227 = 683$

$x + 248 - 53 - x + 412 - 119 - x + 364 = 683$

$852 - x = 683$

$x = 169$

The region of the Venn diagram which includes boys who wear sneakers but not hats is the shaded region in the diagram →



$= 412 - (169 + 35 + 84)$
$= 412 - 288$
$= 124$

CATEGORY 2

① 810,000
② 12.1
③ 2590.5

① Convert 25 feet to inches: $25 \times 12 = 300$ inches
and 100 feet to inches: $100 \times 12 = 1200$ inches.
Volume = (length) · (width) · (height)
$= 1200 \cdot 300 \cdot 2\frac{1}{4}$
$= 810,000$

② Finding the surface area of the golf ball is a bit tricky: Find the surface area as though it were a smooth sphere, then subtract the area of the 234 dimples, then add the surface area of the 234 hemispheres!

SOLUTION KEY - March, 1996

Continued...

② $SA = 4\pi r_1^2 - 234\pi r_2^2 + 234(2\pi r_2^2)$

$= 4\pi r_1^2 - 234\pi r_2^2 + 468\pi r_2^2$

$= 4\pi r_1^2 + 234\pi r_2^2$

$\approx 4(3.14)(0.7)^2 + 234(3.14)(0.09)^2$

$\approx 6.1544 + 5.951556$

≈ 12.105956

≈ 12.1 (nearest tenth)

③ Volume $= \pi r^2 l$, where l is the length of the tunnel, and r is its radius.

$\approx 3.14(4.5)^2(1100)$

$\approx 69{,}943.5$ cubic feet.

The question asks for cubic yards, however!

1 cubic yard = 27 cubic feet.

$69{,}943.5 \div 27 = 2590.5$ cubic yards.

CATEGORY 3

① 30,000

② 8.39

③ 18

① The vertical scale is in thousands of cars!

$90{,}000 - 60{,}000 = 30{,}000$

② during the day: $37(0.43) + 0.59 = 16.50$

during the evening: $37(0.21) + 0.34 = 8.11$

Difference $8.39

③ Let x = my age in years

$5(7) + 7(3) + x = 2x - 20 + 6(9) + 4$

$35 + 21 + x = 2x - 20 + 54 + 4$

$56 + x = 2x + 38$

$18 = x$

CATEGORY 4

① $\frac{1}{10}$

② 94.5

③ $\frac{1}{7}$

① $40\% \times 25\% = .4 \times .25 = .1 = \frac{1}{10}$

② If the average of all seventeen test scores is to be 82, then their total would be 82×17, or 1394. The total of the first fifteen scores is 1205. The difference is $1394 - 1205$, or 189, which is the sum of the two make-up scores. $189 \div 2 = 94.5$.

continued ...

③ This problem is equivalent to the "urn problem without replacement."
The probability that the first chocolate chosen is dark $= \frac{6}{15}$, and that the second one chosen is also dark $= \frac{5}{14}$. $\frac{6}{15} \cdot \frac{5}{14} = \frac{\cancel{2}^{1}\cancel{6}}{\cancel{15}_{5}} \cdot \frac{\cancel{5}^{1}}{\cancel{14}_{7}} = \frac{1}{7}$

CATEGORY 5

① -14 and 3 (any order)

② 216.25

③ 278.3

216.25

① $N^2 + 11N = 42$ can be solved by (A) factoring, (B) Completing the square, or (C) quadratic formula:

(A) $N^2 + 11N = 42$

$N^2 + 11N - 42 = 0$

$(N+14)(N-3) = 0$

$N+14 = 0$ or $N-3 = 0$

$N = -14$ or $N = 3$

(B) $N^2 + 11N = 42$

$N^2 + 11N + \left(\frac{11}{2}\right)^2 = 42 + \left(\frac{11}{2}\right)^2$

$N^2 + 11N + \frac{121}{4} = 42 + \frac{121}{4}$

$\left(N + \frac{11}{2}\right)^2 = \frac{168}{4} + \frac{121}{4}$

$\left(N + \frac{11}{2}\right)^2 = \frac{289}{4}$

$\sqrt{\left(N + \frac{11}{2}\right)^2} = \sqrt{\frac{289}{4}}$

$\left|N + \frac{11}{2}\right| = \frac{17}{2}$

$N + \frac{11}{2} = \pm \frac{17}{2}$

$N = -\frac{11}{2} \pm \frac{17}{2}$

$N = \frac{-11+17}{2}$ or $\frac{-11-17}{2}$

$= \frac{6}{2}$ or $-\frac{28}{2}$

$= 3$ or -14

(C) $N^2 + 11N = 42$

$N^2 + 11N - 42 = 0$

$a = 1$

$b = 11$

$c = -42$

$N = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-11 \pm \sqrt{11^2 - 4(1)(-42)}}{2(1)}$

$= \frac{-11 \pm \sqrt{121 + 168}}{2}$

$= \frac{-11 \pm \sqrt{289}}{2}$

$= \frac{-11 \pm 17}{2}$

$= \frac{-11+17}{2}$ or $\frac{-11-17}{2}$

$= 3$ or -14

...tegory 5, continued ...

(3) $d = rt - 5t^2$

$0 = 65t - 5t^2$

$0 = 5t(13 - t)$

$5t = 0$ or $13 - t = 0$

$t = 0$ or $t = 13$

$d = 0$! The ball was caught at the same height it was thrown.

∴ The maximum height was reached at the halfway point, or after 6.5 seconds:

$d = rt - 5t^2$

$d = 65(6.5) - 5(6.5)^2$

$d = 422.5 - 5(42.25)$

$d = 422.5 - 211.25$

$d = 211.25$ feet Now add 5 ft. (He released the ball from 5 feet above the ground.)

(3) The ball hit the ground 6 feet <u>below</u> its starting point, so $d = -6$:

$d = rt - 5t^2$

$-6 = 74.6t - 5t^2$

$5t^2 - 74.6t - 6 = 0$

Using the quadratic formula, let $a = 5$, $b = -74.6$, and $c = -6$:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{74.6 \pm \sqrt{(-74.6)^2 - 4(5)(-6)}}{2(5)}$$

$$= \frac{74.6 \pm \sqrt{5565.16 + 120}}{10}$$

$$= \frac{74.6 \pm \sqrt{5685.16}}{10}$$

$$= \frac{74.6 \pm 75.4}{10}$$

$$= \frac{74.6 + 75.4}{10} \text{ or } \frac{74.6 - 75.4}{10}$$

$$= \frac{150}{10} \text{ or } \frac{-0.8}{10}$$

$= 15$ or -0.08

(continued)

SOLUTION KEY - March, 1996

Category 5 - continued again ...

③ (continued)

The maximum height of the ball occured at a point halfway between $t = 15$ and $t = -0.08$, or $\frac{15 + (-0.08)}{2} = \frac{14.92}{2} = 7.46$ sec.

$d = rt - 5t^2$

$d = 74.6(7.46) - 5(7.46)^2$

$d = 556.516 - 5(55.6516)$

$d = 556.516 - 278.258$

$d = 278.258$

$d \approx 278.3$ feet (nearest tenth of a foot!)

CATEGORY 6

① 555

② 14

③ 142

④ 129

⑤ 2940

⑥ 46

① $\frac{W}{75} = \frac{1509.6}{(\frac{3}{4})(.8)(340)}$

$\frac{W}{75} = \frac{1509.6}{204}$

$204W = 75(1509.6)$

$204W = 113220$

$W = \frac{113220}{204}$

$W = 555$

② A cyclist's speed without wind = C, the speed of the wind = W :

	Rate x Time = Distance	
with the wind	C+W	$\frac{1}{4}$	4
against the wind	C-W	$\frac{1}{3}$	4

(15 min. = $\frac{1}{4}$ hr)
(20 min. = $\frac{1}{3}$ hr)

$\frac{1}{4}(C+W) = 4$
$\frac{1}{3}(C-W) = 4$

$\rightarrow$

$C + W = 16$
$C - W = 12$

$2C = 28$
$C = 14$

∴ The cyclist's speed is 14 Km per hour.

SOLUTION KEY - March, 1996

③ Let C = # of children
A = # of adults

$$\begin{cases} 4.25C + 7.50A = 1859.75 \\ C + A = 329 \longrightarrow A = 329 - C \end{cases}$$

$$\therefore 4.25C + 7.50(329 - C) = 1859.75$$
$$4.25C + 2467.5 - 7.5C = 1859.75$$
$$-3.25C + 2467.5 = 1859.75$$
$$-3.25C = -607.75$$
$$C = \frac{-607.75}{-3.25}$$
$$C = 187$$
$$\therefore A = 329 - C$$
$$= 329 - 187$$
$$= 142$$

∴ There were 142 adults.

④ Perimeter $= \frac{300}{360}(2\pi r) + 2(18)$

$$\approx \frac{5}{6}(2)(3.1)(18) + 36$$
$$\approx 93 + 36$$
$$\approx 129$$

⑤ Let x = the length of the shorter leg
$2x-7$ = the length of the longer leg
$2x-7+14$ = the length of the hypotenuse

Use the Pythagorean Theorem:

$$x^2 + (2x-7)^2 = (2x-7+14)^2$$
$$x^2 + 4x^2 - 28x + 49 = (2x+7)^2$$
$$5x^2 - 28x + 49 = 4x^2 + 28x + 49$$
$$x^2 - 56x = 0$$
$$x(x-56) = 0$$
$$x = 0 \quad \text{or} \quad x = 56$$
(discard)

SOLUTION KEY - March, 1996

Category 6 - continued ...

So, $x = 56$

$2x - 7 = 105$

$2x - 7 + 14 = 119$

The two legs form the two perpendicular sides of the right triangle, and are the base and height.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}(56)(105)$$
$$= 2940 \text{ in.}^2$$

⑥
$$\frac{C + D + S}{\sqrt[3]{\frac{E}{B} + \sqrt{\frac{A}{15}} - 1}}$$

$$= \frac{142 + 129 + 5}{\sqrt[3]{\frac{2940}{14} + \sqrt{\frac{555}{15}} - 1}}$$

$$= \frac{276}{\sqrt[3]{210 + \sqrt{37} - 1}}$$

$$= \frac{276}{\sqrt[3]{210 + \sqrt{36}}}$$

$$= \frac{276}{\sqrt[3]{210 + 6}}$$

$$= \frac{276}{\sqrt[3]{216}}$$

$$= \frac{276}{6}$$

$$= 46$$