CATEGORY 3 - Mystery
February, 1996 - Meet #4
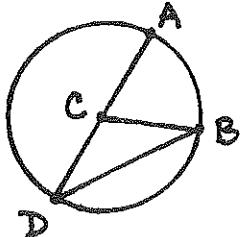
① The natural number, N, gives a remainder of 2 when it is divided by 6. The natural number, M, gives a remainder of 5 when it is divided by 6. What is the remainder when the sum of N and M is divided by 6?

② A Taxi company charges $1.75 for the first mile, and $0.35 for each quarter mile or portion of a quarter mile thereafter. How much would a ride of 23.6 miles from Reading to Marblehead cost?

③ The product of two consecutive, WHOLE numbers is 6972. What is the sum of those two consecutive numbers?
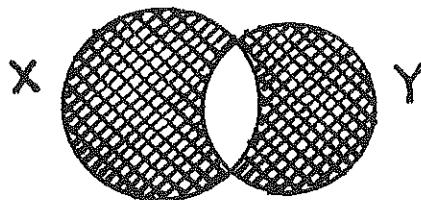
CATEGORY 2 - Geometry
February, 1996 - Meet #4

① Using $\pi \approx 3.14$, the area of a circle is 91.5624 square feet. Find the circumference of the circle, in feet.

② The measure of arc AB is 78°. Find the measure of angle CBD. Point C is the center of the circle, and $\overline{DA}$ is a diameter.



③ The diameter of circle X is 8.6 centimeters. Using $\pi \approx 3.142$, the circumference of circle Y is 21.0514 centimeters. Find the area of the shaded region, which is where the two circles do not intersect, in square centimeters, if the area of the unshaded region is 17.62084 square centimeters. Round your final answer to the nearest hundredth.



ANSWERS

① _____ feet

② _____ degrees

③ _____ sq. cm

① Term # → 1st 2nd 3rd 4th 5th ...

Value → 17 24 31 38 45 ...

Find the value of the 195th term in the sequence above.

② Jack Koo Stowe wanted to break the world's record for holding his breath under water. To do so, he would need to hold his breath for 873 seconds. If the clock read 4:56:47 P.M. when he started holding his breath, then what time would the clock read when Jack breaks the world's record? (Standard notation for time - for A:B:C, A<24, B<60, and C<60. Also include either A.M. or P.M. in your answer.)

③ Find the sum of the first 70 counting numbers
$(1+2+3+ ... + 68+69+70)$.

ANSWERS

① _____

② _____

③ _____

CATEGORY 4 – Arithmetic
February, 1996 – Meet #4

① Mun E. Bags went to the store and purchased these items:

| Item | # of items | cost per item |
|------|-----------|---------------|
| pen | 3 | 79¢ |
| notebook | 4 | $1.58 |
| CD | 7 | $9.75 |

The cost of these items was subtotalled, and then a 5% sales tax was calculated. What was the total cost, including sales tax? Round to the nearest cent.

② Ed lost his job as a shoe salesman, and went looking for another job. The It Fitz Wright shoe store was willing to pay him $4.25 per hour for an 8-hour work day, while the Toe 'n Heel store offered him $1.50 per hour, plus 12% commission based on the total retail value of pairs of shoes sold, plus 15% commission based on the total retail value of pairs of sneakers sold. Ed was told by the manager of Toe 'n Heel that, if he worked hard, he could sell the following on an average day:

| type of footwear | average retail value, per pair | average number of pairs sold |
|------------------|-------------------------------|------------------------------|
| sneakers | $39.95 | 3 |
| shoes | $43.79 | 2 |

Rounded to the nearest cent, how much more would Ed earn per day (8 hours) if he accepted the job at Toe 'n Heel instead of at It Fitz Wright?

③ Find Cory's final bank balance if she started with $364.29, and earned 2¾% interest (annual), compounded every 4 months, and made no deposits or withdrawals for two years. Round to the nearest cent after each calculation.

ANSWERS

① $ _____

② $ _____

③ $ _____
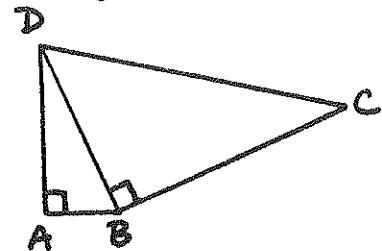
① There were 37 animals in the barnyard, all chickens and pigs. If you counted just the animals' legs, there were 104 in all. How many pigs were there?

② James counted 22 blue M+Ms in his 3.75-ounce bag of M+Ms. To the nearest whole M+M, how many blue M+Ms should James expect to find in a five pound bag, if there is a proportional number of blue M+Ms in bags of all sizes? (16 ounces = 1 pound)

③ Todd and Dott are ready to race around a 440-yard oval track. 440 yards = 1 lap. They start at the same place at the same time. Todd's average speed is 4 yards per second, while Dott's is 6 yards/sec. After how many minutes will Dott be one full lap ahead of Todd? Express your answer as a mixed numeral in simplest form.

ANSWERS

① _____

② _____

③ _____

CATEGORY 6 - Team Questions
February, 1996 - Meet #4

① 90 miles per hour is the same speed as W feet per second. Find the value of W.

② Together, Jan and Dean had 120 baseball cards. Jan gave 20 cards to Dean, so that Dean finally had 26 more cards than Jan. How many cards did Dean have to start with?

③ The mean of a set of five numbers is 39, and the median is 40. If the mean of the two largest numbers is 49, then what is the sum of the two smallest numbers?

④ Find the number of square inches in the area of polygon ABCD. All measurements are in inches. $AD = 8''$, $DC = 26''$, $AB = 6''$:



⑤ XY is a meter stick divided into centimeters, with point X at zero and point Y at 100. At which centimeter mark should point W be located so that the ratio of XW to WY is 17:8?

⑥ Using the answers from #1-5, evaluate the following:

$$\left( D - \left[ (E - B)^2 - 5C + E + A \right] \right)^5$$

ANSWERS

① _____ = A
② _____ = B
③ _____ = C
④ _____ = D
⑤ _____ = E
⑥ _____

# February 1996 – Meet #4 Answers

**3 – Mystery** (Categories used to be numbered differently)
1) 1                    2) 33.60 or 33.6          3) 167

**2 – Geometry**
1) 33.912              2) 39                     3) 58.11

**1 – Number Theory**
1) 1375                2) 5:11:20 PM             3) 2485

**4 – Arithmetic**
1) 80.79               2) 6.49                   3) 384.79

**5 – Algebra**
1) 15                  2) 469                    3) 3 2/3 (three and two-
thirds)

**6 – Team**
1) 132
2) 53
3) 57
4) 144
5) 68
6) 1024

## CATEGORY 1

① 1375

② 5:11:20
P.M.

③ 2485

① Each term is 7 more than the one before it, so the 195th term is 10 more than the 195th multiple of 7, since each term is 10 more than the corresponding multiple of 7.

$$195(7) + 10 = 1375.$$

② First convert 873 seconds to minutes and seconds:
$873 ÷ 60 = 14$ with remainder 33, so 873 sec. = 14 min., 33 sec. Now add to the starting time:
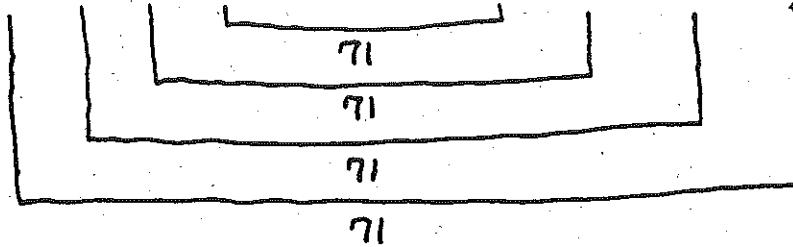
$$\begin{array}{llll} \text{starting time} \rightarrow & 4:56:47 \\ \text{add time holding breath} \rightarrow & \phantom{4}:14:33 \\ \hline & 4:70:80 \end{array}$$

Now simplify $\longrightarrow = 4:71:20$

$$= 5:11:20$$

③ Add in pairs, starting with the extremes:

$$1 + 2 + 3 + 4 + \ldots + 67 + 68 + 69 + 70$$

71

71

71

71

There are 35 sums of 71. $35 \times 71 = 2485.$

## CATEGORY 2

① 33.912

② 39

③ ~~75.73835~~
58.11

① First, work backwards to find the radius:

$$A = \pi r^2$$
$$91.5624 = 3.14 \cdot r^2$$
$$\frac{91.5624}{3.14} = r^2$$
$$29.16 = r^2$$
$$\sqrt{29.16} = r$$
$$5.4 = r$$

Now $C = 2\pi r$
$$= 2(3.14)(5.4)$$
$$= 33.912$$

② $m\angle ACB = 78°$ (A central angle is equal in degrees to its intercepted arc.)
$m\angle DCB = 102°$ (supplement of $\angle ACB$)
$m\angle ADB = 39°$ (An inscribed angle is half the number of degrees of its intercepted arc.)
$m\angle DCB + m\angle ADB + m\angle CBD = 180$ (sum of angles of a $\triangle = 180°$).
$102 + 39 + m\angle CBD = 180$
$m\angle CBD = 39$

Category 2 - continued

③ radius of circle $X$ = 4.3 cm ($\frac{1}{2}$ of diameter of 8.6)

area of circle $X$ = $\pi r^2$

$\qquad$ = $3.142(4.3^2)$

$\qquad$ = 58.09558

radius of circle $Y$ = $21.0514 \div 3.142 \div 2$ $\left(\dfrac{circumference}{2\pi}\right)$

$\qquad$ = 3.35 cm

area of circle $Y$ = $\pi r^2$

$\qquad$ = $3.142(3.35^2)$

$\qquad$ = 35.261095

area of shaded region = area of $X$ + area of $Y$ – $_\wedge$area of unshaded region ^TWICE THE

$\qquad$ = 58.09558 + 35.261095 – 2(17.62084)

$\qquad$ = ~~7·~~ cm² $\quad$ 58.11 cm²

$\qquad$ ∴ ~~58.114995~~

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**CATEGORY 3**

① 1

② 33.60
↗ zero is optional

③ 167

① N is 2 more than a multiple of 6, while M is 5 more than a multiple of 6. N+M is 7 more than a multiple of 6, or 1 more than the next consecutive multiple of 6.

② First mile $\longrightarrow$ = 1.75

next 22 miles) = 88 quarter miles, so $88 \times 0.35 = 30.80$

final 0.6 mile = 2 quarter miles, so $2 \times 0.35 = 0.70$

and 1 portion of a quarter mile = 0.35

$\qquad$ Total $33.60

③ To approximate the value of one #, find $\sqrt{6972} \approx 83.5$.

$83 \times 84 = 6972$.

The question asks for the sum: $83 + 84 = 167$.

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**CATEGORY 4**

① 80.79

② 6.49

③ 384.79

① $3 \times 0.79 =$ $2.37

$\quad 4 \times 1.58 =$ $\quad$ 6.32

$\quad 7 \times 9.75 =$ $\quad$ 68.25

subtotal $76.94

Sales tax: $.05 \times 76.94 = 3.847$

$\qquad \approx 3.85$

plus subtotal $\quad + \ 76.94$

$\qquad$ Total $\quad$ $80.79

② Toe 'n Heel:

salary – $8 \times 1.50 =$ $12.00

commission –

$\quad$ sneakers:

$\quad .15 \times (3 \times 39.95) \approx$ 17.98

$\quad$ shoes:

$\quad .12 \times (2 \times 43.79) \approx$ 10.51

$\qquad$ Total $40.49

It Fitz Wright: $8 \times 4.25$

$\qquad$ = $34.00

Difference: $\quad 40.49 - 34.00$

$\qquad$ = $6.49

SOLUTION KEY – Meet #4
February, 1996

Category 4 - continued

③ Interest will be compounded six times). Start with #364.29

Interest after 4 months : $364.29 \times .0275 \div 3 \doteq 3.34$

balance after 4 months = 367.63

Interest after 8 months : $367.63 \times .0275 \div 3 \approx 3.37$

balance after 8 months = 371.00

Interest after 12 months : $371.00 \times .0275 \div 3 \approx 3.40$

balance after 12 months = 374.40

Interest after 16 months : $374.40 \times .0275 \div 3 \approx 3.43$

balance after 15 months = 377.83

Interest after 20 months : $377.83 \times .0275 \div 3 \approx 3.46$

balance after 20 months = 381.29

Interest after 24 months : $381.29 \times .0275 \div 3 \approx 3.50$

final balance after 24 months = 384.79

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CATEGORY 5

① 15

② 469

③ $3\frac{2}{3}$

① Let $x$ = # of chickens
    $y$ = # of pigs

$\begin{cases} x+y = 37 \\ 2x+4y = 104 \end{cases}$  (There are 37 animals in all.)
(Each chicken has two legs, pigs have four)

$\begin{cases} 2x+2y = 74 \\ 2x+4y = 104 \end{cases}$  (Mult. both members of first equation by 2)
(second equation)

$2y = 30$  (Subtracting equation 1 from equation 2)
$y = 15$

② $\dfrac{\text{\# of M+Ms}}{\text{\# of ounces}}$ :  $\dfrac{22}{3.75} = \dfrac{x}{80}$  (Five pounds contains
5×16, or 80 ounces.)

$3.75x = 22(80)$   Cross-products are equal.
$3.75x = 1760$
$x = \dfrac{1760}{3.75}$
$x \approx 469.3$
$x \approx 469$   (rounded to whole m+m)

③ Let $x$ = the # of seconds each person runs
    $4x$ = Todd's distance
    $6x$ = Dott's distance
Use the idea that Dott runs 440 yards farther than Todd.

Category 5 - continued

$4x + 440 = 6x$        Now, for the answer:
$\phantom{4x +}440 = 2x$        220 seconds $= 3\frac{40}{60}$ minutes
$\phantom{4x + 4}220 = x$        $\phantom{220 seconds } = 3\frac{2}{3}$ minutes.

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**CATEGORY 6**

① 132
② 53
③ 57
④ 144
⑤ 68
⑥ 1024

① $\dfrac{90 \text{ miles}}{\text{hour}} \times \dfrac{5280 \text{ feet}}{1 \text{ mile}} \times \dfrac{1 \text{ hour}}{60 \text{ min}} \times \dfrac{1 \text{ min.}}{60 \text{ sec.}}$

Now "cancel" like crazy!

$\dfrac{\cancel{90} \text{ miles}}{\cancel{\text{hour}}} \times \dfrac{\cancel{5280} \text{ feet}}{\cancel{1 \text{ mile}}} \times \dfrac{1 \text{ hour}}{\cancel{60} \text{ min.}} \times \dfrac{1 \text{ min.}}{\cancel{60} \text{ sec.}} = \dfrac{132 \text{ feet}}{1 \text{ sec.}}$

② 
|  | before | after |
|------|------|------|
| Jan | $x$ | $x-20$ |
| Dean | $120-x$ | $120-x+20$ |

$x - 20 + 26 = 120 - x + 20$
$\phantom{x - 20}x + 6 = 140 - x$
$\phantom{x - 20 + }2x = 134$
$\phantom{x - 20 + 2}x = 67$
$120 - x = 53$

③ If the mean is 39, then the sum of the five numbers is $39 \times 5$, or 195. If the mean of the two largest is 49, then their sum is 98. The sum of the three largest numbers is $40 + 98$, or 138. So the sum of the two smallest is $195 - 138$, or 57.

④ Use the Pythagorean Theorem, twice, to find the measures of $\overline{BD}$ and $\overline{BC}$, which are the base and height of $\triangle DBC$:

$8^2 + 6^2 = (BD)^2$        $10^2 + (BC)^2 = 26^2$        area $(\triangle ABD) = \frac{1}{2}(6)(8)$
$64 + 36 = (BD)^2$        $100 + (BC)^2 = 676$        $\phantom{area (\triangle ABD)} = 24 \text{ in}^2$
$\phantom{64 + }100 = (BD)^2$        $\phantom{100 + }(BC)^2 = 576$        area $(\triangle DBC) = \frac{1}{2}(10)(24)$
$\phantom{64 + 10}10 = BD$        $\phantom{100 + }BC = 24$        $\phantom{area (\triangle DBC)} = 120 \text{ in}^2$

∴ The area of polygon ABCD $=$ area $(\triangle ABD) +$ area $(\triangle DBC)$
$\phantom{∴ The area of polygon ABCD } = 24 + 120$
$\phantom{∴ The area of polygon ABCD } = 144 \text{ in}^2$

Category 6 - continued

⑤ Since $17+8 = 25$, and $25 \times 4 = 100$, this scaling is quick and simple! $17 \times 4 = 68$, $8 \times 4 = 32$, $68 + 32 = 100$.
∴ W should be located at the 68 cm mark.

⑥
$$\left( D - \left[ (E-B)^2 - 5c + E + A \right] \right)^5$$

$$= \left( 144 - \left[ (68-53)^2 - 5(57) + 68 + 132 \right] \right)^5$$

$$= \left( 144 - \left[ 15^2 - 285 + 200 \right] \right)^5$$

$$= \left( 144 - \left[ 225 - 285 + 200 \right] \right)^5$$

$$= \left( 144 - 140 \right)^5$$

$$= 4^5$$

$$= 1024$$