CATEGORY 1 - NUMBER THEORY
MARCH, 1995 - MEET #5

YOU MAY USE A CALCULATOR IN ALL CATEGORIES TODAY!

① If $A = \{4, 8, 12, 16, \ldots\}$
$B = \{6, 12, 18, 24, 30\}$
$C = \{3, 6, 9, \ldots\}$ then find $(A \cap B) \cap C$.

② If $N = \{$prime numbers between 130 and 140$\}$
$W = \{$multiples of 3 between 130 and 140$\}$
$X = \{$multiples of 7 between 130 and 140$\}$
then find $(N \cup W) \cap (N \cup X)$.

③ Of 14,294 students surveyed at UMass - Amherst,
6849 watch "Seinfeld" on TV,
4235 watch only "Seinfeld",
3807 watch "Frasier",
1851 watch "Frasier" and "Seinfeld",
9371 watch either "Lois and Clark" or "Seinfeld", but not both shows,
1628 watch all three shows.

If L = the total number of students who watch "Lois and Clark", and
F = the number of students who just watch "Frasier",
then calculate L - F.

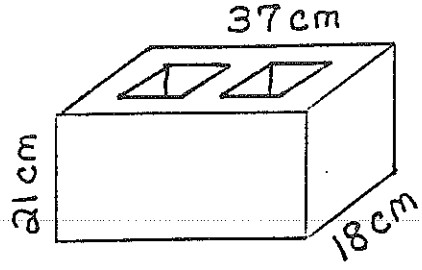
ANSWERS

① { }

② { }

③ _____

CATEGORY 2 - GEOMETRY
MARCH, 1995 - MEET #5

① 

A cement block has two holes going completely through it. Each hole measures 13 cm by 15 cm at the top of the block. How many cubic centimeters of cement were used to make the block pictured?

② How many square centimeters are in the surface area of the block pictured above, if the surface area consists of all exposed surfaces?

③ A solid rubber bowling ball, 10.3 inches in diameter, has three cylindrical holes drilled - two to make room for fingers, each 3.4 in. deep and 1.2" in diameter, and one to make room for a thumb, which is 4.6 in. deep and 1.7 in. in diameter. The ball is packed tightly into a box so that it touches the inside of the box on all sides. None of the holes in the ball touch an inner surface of the box. To the nearest tenth of a cubic inch, what is the volume of the airspace inside the box? Use $\pi \approx 3.14$. Do no intermediate rounding - round off only your final answer.

ANSWERS

① _____ cm^3

② _____ cm^2

③ _____ $in.^3$

CATEGORY 3 - MYSTERY
MARCH, 1995 - MEET #5

① Jaug Phast is running around a $\frac{1}{4}$-mile track at 9 miles per hour. How many times did she run around the track if she ran for 35 minutes?

② A paper manufacturer produces 326,947 sheets of paper in one day. If he wishes to box as many sheets of paper as possible, and there are 85 sheets per pad, and 22 pads to a box, then how many sheets of paper are unboxed each day?

③ Yoko called her aunt in Japan. The call cost #0.92 per minute for the first three minutes, and #0.76 for each additional minute or part of a minute. What is the greatest whole number of minutes that Yoko could talk to her aunt and pay no more than #16?

ANSWERS

① _____

② _____

③ _____

CATEGORY 4 – ARITHMETIC
MARCH, 1995 – MEET #5

① A box of chocolates contains 15 nut centers, 12 caramels, 7 fruit centers, and 8 surprise centers. If a chocolate is chosen at random, what is the probability that it is either a caramel or a surprise center? Express your answer as a simplified fraction.

② The mode of a set of nine numbers is 25, while the mean is 29. What is the median of the nine numbers if seven of the nine numbers are 25, 38, 30, 43, 25, 30, and 28?

③ The probability that it will snow this Saturday is 40%. The probability that it will snow this Sunday is 25%. What is the probability that it will snow sometime this weekend (Saturday or Sunday)? Express your answer as a percent.

ANSWERS

① _____

② _____

③ _____ %

CATEGORY 5 – ALGEBRA
MARCH, 1995 – MEET #5

① Find both values of N which make this equation true:

$$N^2 - 24 = -5N$$

② If an object is thrown (or kicked, or shot, etc.) into the air with an initial upward velocity of R meters per second, then its distance, D meters, above its starting point at time T seconds after it was thrown is approximately

$$D = RT - 5T^2$$

A ball is thrown into the air with an initial upward velocity of 55 meters per second. How many seconds after it is thrown will it first reach a height of 90 meters?

③ Refer to problem #2. How many seconds after it reaches its maximum (highest) height will the ball again be 90 meters above the ground? Express your answer as a decimal.

ANSWERS

① { }

② _____ sec.

③ _____ sec.

CATEGORY 6 - TEAM QUESTIONS
MARCH, 1995 - MEET #5

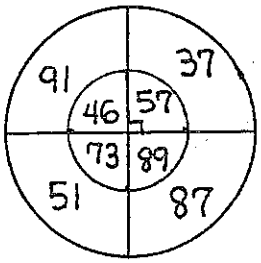
① If $N^A = 262{,}144$, and $N \leq 18$, and $A \leq 8$, then find the sum $N + A$.

② Consider a "word" to be an arrangement of letters, whether or not those letters actually spell a real word. How many different "words" can be formed using the letters in the word COTTON?

③ A rectangular box has a length of 15.6 inches, a width of 12.7 inches, and a volume of 1862.328 cubic inches. What is the surface area, in square inches, of the largest spherical ball which will fit into the closed box? Use $\pi \approx 3.14$. Round your answer to the nearest whole numbers.

④ Find the largest value of Y which makes $Y^2 + 18Y = 144$ a true statement.

⑤



The target at the left is to be used in a dart game. The smaller circle has a diameter which is half the diameter of the larger circle. A dart is thrown and hits the target. The thickness of the lines is negligible (can be ignored as unimportant). Let P = the probability, as a simplified fraction that the dart hits an area containing a prime number. Give the value of the numerator of P.

⑥ Using the answers to #1-5 above, evaluate:

$$\sqrt[E]{5C^2 - (11BD + 33A + 10)}$$

ANSWERS

① _____ = A
② _____ = B
③ _____ = C
④ _____ = D
⑤ _____ = E
⑥ _____

Solution Key - March, 1995

CATEGORY 1

① 12, 24

② 131
137
139
in any order

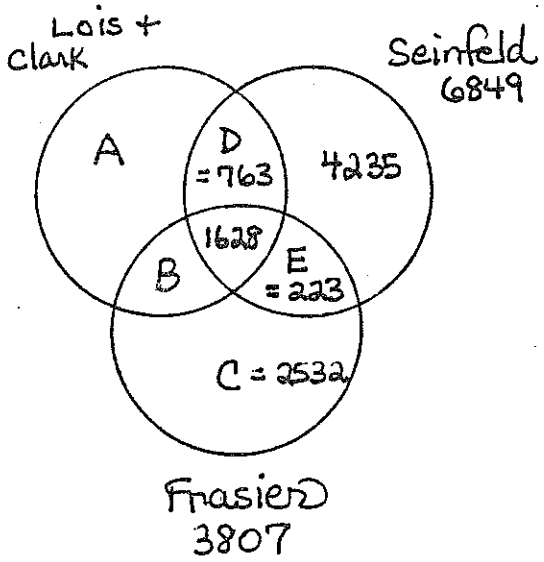
③ 4772

① Since A contains 24 (a multiple of 4), then $A \cap B = \{12, 24\}$. C contains 12 and 24, so $(A \cap B) \cap C = \{12, 24\}$.

② $N = \{131, 137, 139\}$ $W = \{3, 6, 9, \ldots, 132, 135, 138, \ldots$
$X = \{7, 14, 21, \ldots, 133, \ldots\}$

$$(N \cup W) \cap (N \cup X)$$
$$= \{131, 132, 135, 137, 138, 139\} \cap \{131, 133, 137, 139\}$$
$$= \{131, 137, 139\}$$

③ Make a Venn diagram:



To get E, use clue #4:
$E + 1628 = 1851$
$\therefore E = 223$

To get D, use clue #1, 2, 4,
$D + 1628 + 223 + 4235 = 6849$
$\therefore D = 763$

Clue #5 helps find A+B:
$A + B + 4235 + 223 = 9371$
$\therefore A + B = 4913$

So, $L = 4913 + 763 + 1628$
$L = 7304$

To find C, subtract the sum of all other parts from 14,294:

$$14294 - (4913 + 763 + 1628 + 4235 + 223)$$
$$= 14{,}294 - (11{,}762)$$
$$= 2532$$

$\therefore L - F = 7304 - 2532$
$= 4772$

1995

CATEGORY 2

① 5796

② 5214

③ 539.0 (Also accept 539)

① The volume of the block = the product of the outer dimensions minus the volume of the holes

$= lwh_{block} - 2lwh_{holes}$

$= 37(18)(21) - 2(13)(15)(21)$

$= 13,986 - 8190$

$= 5796\ cm^3$

② Be careful - the holes produce more surface!

front :	21×37 ⟶	777
back :	21×37 ⟶	777
left :	21×18 ⟶	378
right :	21×18 ⟶	378
top :	$18 \times 37 - 2(13 \times 15)$ ⟶	276
bottom :	$18 \times 37 - 2(13 \times 15)$ ⟶	276
one hole :	$2 \times 15 \times 21 + 2 \times 13 \times 21$ ⟶	1176
other hole :	$2 \times 15 \times 21 + 2 \times 13 \times 21$ ⟶	1176
	Total surface area	$5214\ cm^2$

③ $Vol._{air} = Vol._{box} - (Vol._{ball} - Vol._{holes})$

$= e^3 - \left(\frac{4}{3}\pi r^3 - (2\pi r^2 h_{fingers} + \pi r^2 h_{thu}\right.$

$\doteq (10.3)^3 - \left[\frac{4}{3} \cdot 3.14 \cdot 5.15^3 - (2 \cdot 3.14 \cdot 0.6^2 \cdot 3.4\right.$

$+ 3.14 \cdot 0.85^2 \cdot 4.1$

$\doteq 1092.727 - [571.860\ldots - (7.68672 + 10.4357$

$\doteq 1092.727 - [571.860 - 18.12251]$

$\doteq 1092.727 - 553.73749$

$\doteq 538.98951$

$\doteq 539.0\ in.^3$

This answer is achieveable whether an 8-digit or 10-digit calculator is used.

CATEGORY 3

① 21

② 1567

③ 20

① Let M = # miles that Jaug ran

$$\frac{9}{60} = \frac{M}{35}$$

$$60M = 9 \cdot 35$$

$$60M = 315$$

$$M = 5\tfrac{1}{4}$$

The # of times that Jaug ran around the track is

$$5\tfrac{1}{4} \div \tfrac{1}{4} = 21$$

② The greatest # of boxes used would be

$$\frac{326,947}{85(22)} = \frac{326,947}{1870} = 174.8379\ldots$$

174 boxes can be filled. To find how many sheets don't get boxed,

$$326,947 - (1870)(174) = 1567$$

③ Let C = # of minutes Yoko could talk at #0.76 / minute

$$0.76C + 3(0.92) \leq 16$$

$$0.76C + 2.76 \leq 16$$

$$0.76C \leq 13.24$$

$$C \leq 17.4210\ldots$$

Yoko could talk no more than 17 minutes at #0.76 / minute, plus the 3 minutes at #0.92 / minute, for a total of 20 minutes.

CATEGORY 4

① $\frac{10}{21}$

② 28

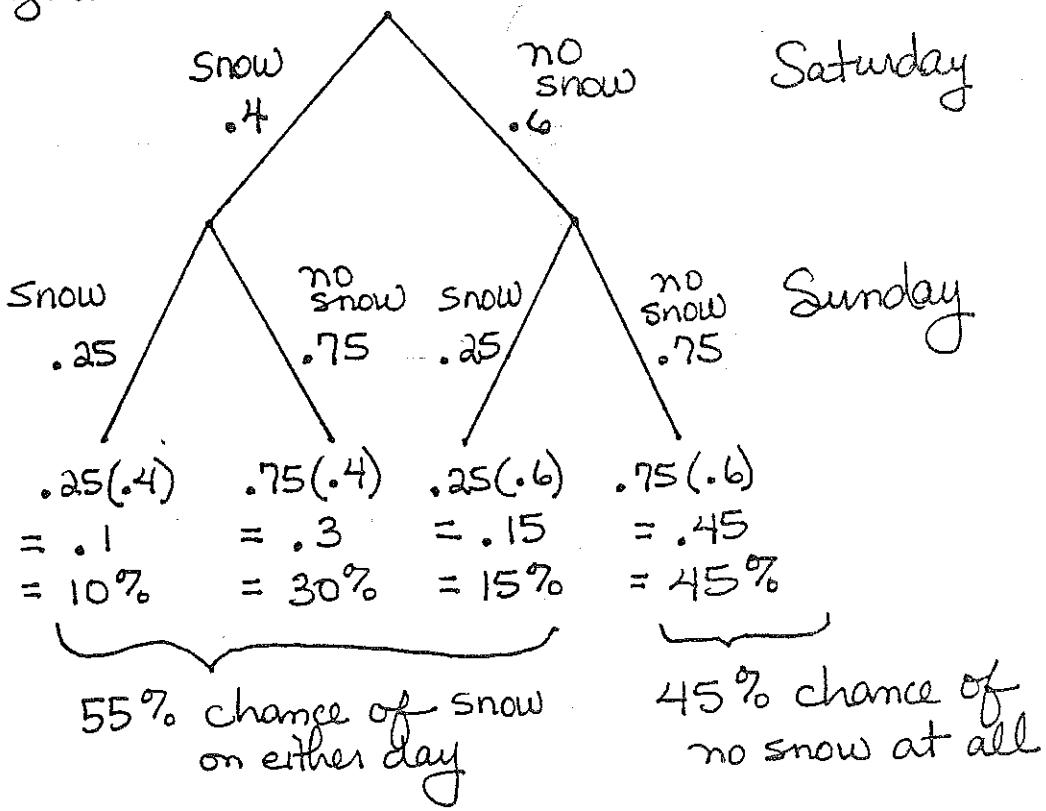
③ 55%

① $\frac{12+8}{42} = \frac{20}{42} = \frac{10}{21}$

② If the mean of the nine numbers is 29, then the sum is 9×29, or 261. The sum of the given numbers is 219, so the sum of the remaining two numbers is $261 - 219$, or 42. If 25 is the mode, then one of the two missing numbers is 25 so there are more 25's than any other number. So the other missing number is $42 - 25$, or 17. To find the median, put the numbers in order:

17 25 25 25 (28) 30 30 38 43

③ To visualize the solution, try using a "tree" diagram:



CATEGORY 5

① $\{3, -8\}$

② 2

③ 3.5 only

(The question asks for a decimal.)

① $N^2 - 24 = -5N$

$N^2 + 5N - 24 = 0$ Standard form

$(N+8)(N-3) = 0$ Factor

$N+8=0$ or $N-3=0$ Zero product theorem

$N = -8$ $N = 3$

② $D = RT - 5T^2$ Formula

$90 = 55T - 5T^2$ Substitute Known values

$5T^2 - 55T + 90 = 0$ Standard form

$T^2 - 11T + 18 = 0$ Divide both members by 5

$(T-9)(T-2) = 0$ Factor

$T-9=0$ or $T-2=0$ Zero product theorem

$T=9$ $T=2$

The ball reaches a height of 90 meters on the way up at 2 seconds, and again on the way down at 9 seconds).

③ The time it takes the ball to reach its maximum height can be found quickly by averaging the two symmetry points which are each 90 m above the ground, namely 2 sec. and 9 sec.: $\frac{2+9}{2} = 5\frac{1}{2}$ sec. It therefore takes an additional $3\frac{1}{2}$ seconds to reach a point 90 m above the ground. $3\frac{1}{2} = 3.5$

CATEGORY 6

① 14

② 180

③ 277

④ 6

⑤ 5

⑥ 13

① Trial and error, using a calculator, may solve this one quickly. $8^6 = 262,144$, so $8+6=14$.

Or, factor 262,144:

$$262,144 = \underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}$$

2^{18} and 4^9 don't work, because the exponents are greater than 8. However, there are 6 factors of 2^3

$$\underbrace{2\cdot2\cdot2}_{8}\cdot\underbrace{2\cdot2\cdot2}_{8}\cdot\underbrace{2\cdot2\cdot2}_{8}\cdot\underbrace{2\cdot2\cdot2}_{8}\cdot\underbrace{2\cdot2\cdot2}_{8}\cdot\underbrace{2\cdot2\cdot2}_{8}$$

So, $262,144 = 8^6$, which satisfies all stated conditions.

CATEGORY 6 (continued)

② $\frac{6!}{2! \cdot 2!}$ ← The total # of "words" assuming all letters are different

↖ accounts for two T's, producing another set of identical "words"

↗ accounts for two O's, producing two sets of identical "words"

$$\frac{6!}{2! \cdot 2!} = \frac{6 \cdot 5 \cdot \cancel{4} \cdot 3 \cdot 2 \cdot 1}{\cancel{2} \cdot 1 \cdot \cancel{2} \cdot 1} = 6 \cdot 5 \cdot 3 \cdot 2 = 180$$

③ First find the height of the box to see if it is the shortest dimension:

$$V = lwh$$
$$1862.328 = 15.6(12.7)h$$
$$h = 9.4$$

The height, 9.4", is the shortest dimension, so the largest ball which will fit into the box has a diameter of 9.4", or a radius of 4.7".

$$\text{Surface area} = 4\pi r^2$$
$$\approx 4(3.14)(4.7^2)$$
$$\approx 277.4504$$
$$\approx 277 \text{ (nearest whole number)}$$

④ $Y^2 + 18Y = 144$

$Y^2 + 18Y - 144 = 0$ Standard form

$(Y+24)(Y-6) = 0$ Factor

$Y+24 = 0$ or $Y-6=0$ Zero product theorem

$Y = -24$ or $Y = 6$

The question asks for the largest value, which is 6.

⑤ The larger circle has four times the area of the smaller circle, since the diameter ratio is 2:1. Students may know this, or could discover this by assigning arbitrary values to the radii in the ratio 2:1. Therefore, any small sector (e.g., the one containing the number 46) is $\frac{1}{4}$ of the area of the larger sector which contains it (e.g., the one containing both the 46 and the 91).

CATEGORY 6 (continued again)

So, the area containing the #91 and the area containing the #46 are in the ratio 3:1. Using these numbers, each region can be represented, by area, as a 3 or as a 1 as follows:

91 - 3
46 - 1
57 - 1
51 - 3
87 - 3
37 - 3
73 - 1
89 - 1

} prime numbers represent a total of 5 area units

16 = total # of area units

Prime numbers represent $\frac{5}{16}$ of the area of the target, so the probability of hitting a prime number is $\frac{5}{16}$. The numerator is 5.

⑥

$$\sqrt[E]{5C^2 - (11BD + 33A + 10)}$$

$$= \sqrt[5]{5 \cdot 277^2 - [(11 \cdot 180 \cdot 6) + 33 \cdot 14 + 10]}$$

$$= \sqrt[5]{5 \cdot 76{,}729 - [(11{,}880) + 462 + 10]}$$

$$= \sqrt[5]{383{,}645 - [11{,}880 + 462 + 10]}$$

$$= \sqrt[5]{383{,}645 - 12{,}352}$$

$$= \sqrt[5]{371{,}293}$$

$$= 13$$