CATEGORY 1 — Number Theory
February, 1995 — Meet #4

①

A "twilight zone" marathon starts at 3 o'clock, as on the mod 8 clock shown to the left. The marathon lasts for 170 hours. At what time is the marathon finished?

Assume that the length of time between two consecutive numbers on the clock is one hour.

② What is the value of the seventh term in the following sequence?

6   7   12   21   34   51   ___

③ What is the sum of the first 240 terms of the following sequence?
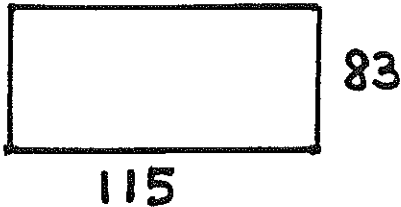
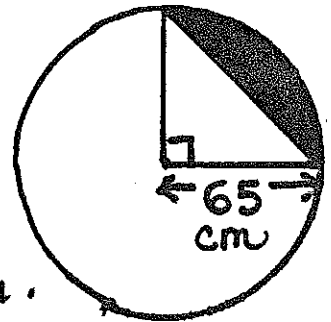5   12   19   26   33   . . .

ANSWERS

① _____ o'clock

② _____

③ _____

CATEGORY 2 - Geometry
February, 1995 - Meet #4

① What is the circumference of the largest circle which will fit inside this rectangle? Use $\pi \approx \frac{22}{7}$. Express your answer as a mixed number in simplest form.

83

115

② Find the area of the shaded region in square centimeters if N is the center of the circle. Use $\pi \approx 3.142$. Round your answer to the nearest whole number.

←65→
cm

③ A goat is free to roam around a backyard, but with one restriction - one end of his 17-foot leash is attached to a corner of a rectangular barn measuring 8 feet by 15 feet. What is the area, in square feet, of the part of the backyard where the goat is able to roam? Use $\pi \approx 3.14$. Round your answer to the nearest whole number.

ANSWERS

① _____

② _____

③ _____

① Reid can read 200 pages in 4 hours. At that rate, how many minutes would it take Reid to read 700 pages?

② Emily sent 75 postcards in five days. Each day she sent six more than on the day before. How many postcards did Emily send on the fourth day?

③ Gummy worms cost 93 cents per dozen, or 9 cents each if purchased separately. If Rebecca buys 75 gummy worms, then what is the largest amount of change that she could receive from a ten dollar bill?

ANSWERS

① _____

② _____

③ _____

CATEGORY 4 - Arithmetic
February, 1995 - Meet #4

① A shirt was originally marked $29.95. It was then discounted 35%. What is the sale price?

② A shoe salesman earns $2.25 per hour at the local shoe store, plus commission. He earns a 5% commission whenever he sells a pair of shoes, and $2.30 for each pair of sneakers sold. Here is a summary of what he sold during his 8-hour shift today:

type	price
shoes	$29.95
shoes	$72.00
sneakers	$42.50
shoes	$56.75
sneakers	$34.00
shoes	$48.30

How much money did he earn today?

③ Sarah has a balance of $265.46 in her savings account, which earns 2% interest annually, compounded quarterly (every three months). If she withdraws no money, and makes no deposits, what will her balance be in 1½ years, if interest is automatically added to her account every three months?

ANSWERS

① # _____

② # _____

③ $ _____

CATEGORY 5 - Algebra
February, 1995 - Meet #4

① Three consecutive multiples of 28 have a sum of 1512. What is the product of the smallest and the largest of the three multiples of 28?

② Two numbers whose sum is 9048 have a difference of 1318. What is the larger of the two numbers?

③ Eric leaves his house, and starts walking at a rate of 35 meters per minute. His brother, Derek, leaves the house one hour later and rides his bike to try to catch up to his brother. Derek's rate is 116 meters per minute. To the nearest minute, how long will it take Derek to catch up to Eric?
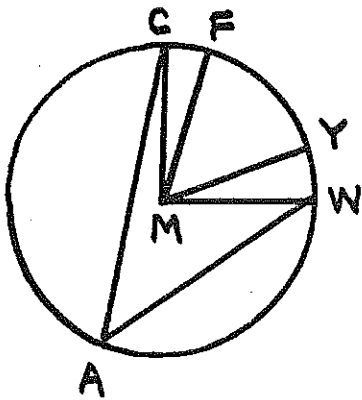
ANSWERS

① _____

② _____

③ _____ min.

① Find the missing number in this number pattern:

8　　11　　17　　29　　54　　103　　___

②

$m\angle CAW = 47°$

$m\angle CMF = 12°$

$m\angle WMY = 17°$

what is the measure of $\angle FMW$?

③ The average of the first six tests is 87. The average of the next three tests is 93. What must the next test score be so that the average of all ten tests is 90?

④ Find the smallest value of N, such that

 $- ⑤ = 568$, if

□ means "square the number inside",

○ means "subtract 3 from the number inside",

△ means "double the number inside".

⑤ Find the sum of all whole numbers from 38 through 113, including 38 and 113.

⑥ Evaluate:

$$\frac{\sqrt{E + D - 10} + 4A + B - 3}{C}$$

ANSWERS

① _____ = A

② _____ = B

③ _____ = C

④ _____ = D

⑤ _____ = E

⑥ _____

Solution Key – February, 1995

① $170 \div 8 = 21$, remainder 2. After 21 rotations, it is again 3 o'clock. Two hours later, it is 5 o'clock

② Look at the sequence of differences :

$$
\begin{array}{ccccccc}
6 & 7 & 12 & 21 & 34 & 51 & \underline{72} \\
& 1 & 5 & 9 & 13 & 17 & 21 \\
& & 4 & 4 & 4 & 4 & 4
\end{array}
$$

③ Each term of this sequence is 2 less than a multiple of 7. So, the 240th term is $240(7) - 2$, or 1678. The 240 terms of the sequence can be arranged in pairs so that the sum of each pair is the same :

$$5 + 12 + 19 + \cdots + 1664 + 1671 + 1678$$

1683
1683
1683

There are 120 such pairs (half of 240), so the sum is $120(1683) = 201,960$.

① The largest circle has a diameter of 83.
$C = \pi d$, or $\frac{22}{7}(83) = \frac{1826}{7} = 260\frac{6}{7}$.

② The shaded area can be found by subtracting the area of the right triangle from the area of the quarter-circle sector which contains it :

$$\frac{1}{4}\pi r^2 - \frac{1}{2}bh = \frac{1}{4}(3.142)(65)^2 - \frac{1}{2}(65)(65)$$
$$= 3318.7375 - 2112.5$$
$$= 1206.2375 \approx 1206$$

③

There are three $\frac{1}{4}$-circles with radii 2, 17, and 9 respectively, and a semi-circle with radius = 17.

$$\frac{1}{4}\pi \cdot 2^2 + \frac{1}{4}\pi \cdot 17^2 + \frac{1}{4}\pi \cdot 9^2 + \frac{1}{2}\pi \cdot 17^2$$
$$= \pi + 72.25\pi + 20.25\pi + 144.5\pi$$
$$= 238\pi \approx 238(3.14) \approx 747.32$$
$$\approx 747$$

CATEGORY 3

① 840

② 21

③ $ 4.15

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① $\dfrac{200 \text{ pages}}{4 \text{ hr.}} = \dfrac{700 \text{ pages}}{14 \text{ hr.}}$

Now convert 14 hours to minutes: $14(60) = 840$

② Trial + error solution:
Try an arithmetic sequence with a difference of 6:

$1 + 7 + 13 + 19 + 25 = 65$,
which is ten short, so add two to each of the five terms.

$3 + 9 + 15 + 21 + 27 = 75$
              ↑
            4th day

Algebraic solution:

Let x = # of cards, day 1
$x+6$ = # of cards, day 2
$x+12$ = " " day 3
$x+18$ = " " day 4
$x+24$ = " " day 5

$x + x+6 + x+12 + x+18 + x+24 =$
$5x + 60 = 75$
$5x = 15$
$x = 3$
$\rightarrow x + 18 = 21$

③ $75 \div 12 = 6$ remainder 3, so 75 worms is 6 dozen plus 3 worms.

$6(.93) + 3(.09) = \$5.58 + \$0.27 = \$5.85$

Change received: $\$10 - 5.85 = \$4.15$

CATEGORY 4

① 19.47

② 32.95

③ 273.52

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① 35% of $\$29.95 = 10.4825 \approx$ $\begin{array}{r} 29.95 \\ -10.48 \\ \hline \$19.47 \end{array}$

② Money earned from: hourly wages $8(2.25) = \$18.0$
                          shoe sales $.05(207.00) = 10.3$
                          sneaker sales $2(2.30) = 4.6$
                                          Total $\$32.9$

③

	Principle · Rate · Time	yearly (years)	(balance 265.46
Interest after 3 months:	$265.46(.02)(.25) \approx$		1.33
		balance	266.79
Interest after 6 months:	$266.79(.02)(.25) \approx$		1.33
		balance	268.12
Interest after 9 months:	$268.12(.02)(.25) \approx$		1.34
		balance	269.46
Interest after 1 year:	$269.46(.02)(.25) =$		1.35
		balance	270.81
Interest after 15 months:	$270.81(.02)(.25) \approx$		1.35
			272.16
Interest after 1½ years:	$272.16(.02)(.25) \approx$		1.36
		balance	273.52

① 253,232

② 5183

③ 26

① Let x = a multiple of 28
  $x+28$ = the next consecutive multiple of 28
  $x+56$ = the next consecutive multiple of 28

  $x + 28 + x + x + 56 = 1512$
  $\qquad 3x + 84 = 1512$
  $\qquad\qquad 3x = 1428$
  smallest → $\qquad x = 476$
  largest → $x+56 = 532$

  smallest × largest
  $= 476 \times 532$
  $= 253,232$

② $x + y = 9048$
   $x - y = 1318$ Add
   ――――――――――
   $2x \quad = 10366$
   $\qquad x = 5183$ ← The question asks for the larger number.
   $\qquad y = 3865$

③ It may be easiest to express all data in terms of minutes :
   Let x = # of minutes since Erik left the house (time until Derek caught up to him).
   $x-60$ = # of minutes Derek is riding his bike until he catches up to Eric

   Their distances are the same :
   $\qquad 35x = 116(x-60)$
   $\qquad 35x = 116x - 6960$
   $\qquad -81x = -6960$
   $\qquad\quad x \approx 85.9259$
   $\qquad\quad x \approx 86$ rounded to nearest minute

   $x-60$
   $= 86-60$
   $= \boxed{26}$

① 201 /97

② 82

③ 99

④ −17

⑤ 11,476

⑥ 10

① Each number in this pattern is 5 less than twice the previous number :
   $8(2) - 5 = 11, \qquad 11(2) - 5 = 17,$ etc.
   $\therefore 103(2) - 5 = 201$

② $\angle A$ is half of angle CMW, so $m\angle CMW = 94°$
   $\angle CMF + \angle FMW = \angle CMW$
   $\qquad 12 + \angle FMW = 94$
   $\qquad\qquad m\angle FMW = 82°$

③ $6(87) = 522$
   $3(93) = 279$
   ――――――
   $\qquad\quad 801$

   For the average of the ten tests to be 90, the total must be $10(90)$, or 900.
   $\qquad 900 - 801 = 99$

④ Translated into an equation using more familiar symbols,

$$2N^2 - 3 - 3 - (5-3)^2 = 568$$
$$2N^2 - 6 - \quad 4 \quad = 568$$
$$2N^2 - 10 \quad = 568$$
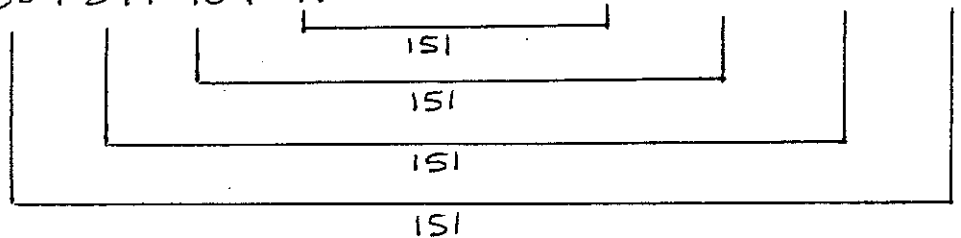$$2N^2 = 578$$
$$N^2 = 289$$
$$N = \pm\sqrt{289}$$
$$N = 17 \text{ or } -17$$

The question asks for the smallest value of N, which is −17.

⑤ There are $(113 - 38 + 1)$, or 76 whole numbers from 38 through 113.
Adding in a special order produces pairs of constant sums :

$$38 + 39 + 40 + 41 + \ldots + 110 + 111 + 112 + 113$$

151

151

151

151

$$76(151) = 11,476$$

⑥

$$\frac{\sqrt{E + D - 10} + 4A + B - 3}{C}$$

$$= \frac{\sqrt{11,476 + (-17) - 10} + 4(201) + 82 - 3}{99}$$

$$= \frac{\sqrt{11,449} + 804 + 82 - 3}{99}$$

$$= \frac{107 + 804 + 82 - 3}{99} \longrightarrow = \frac{990}{99} = 10$$