Category 1
Number Theory
March, 1994

(1 point)

① Change $20121_{\text{Base 3}}$ to a base 10 numeral.

(2 points)

② Simplify $\dfrac{0.00297 \times 10^{42}}{900 \times 10^{15}}$ and express your answer in scientific notation.

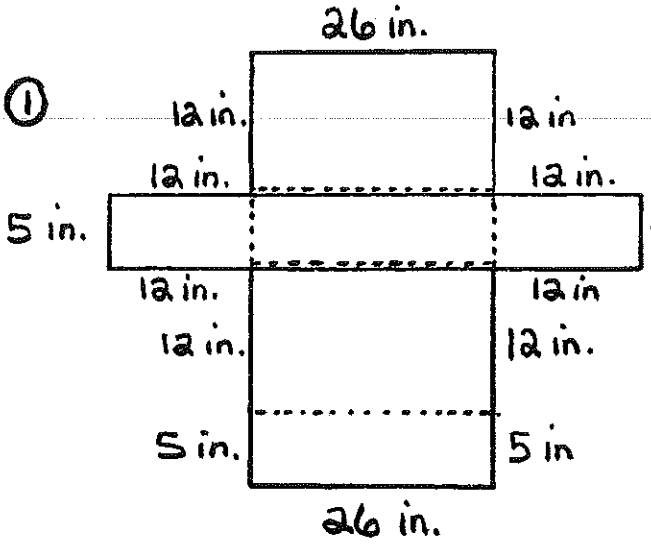
(3 points)

③ Change $734_{\text{Base 9}}$ to a base 4 numeral.

ANSWERS

1. _____
2. _____
3. _____

Category 2
Geometry
March, 1994

① 

The figure at the left is folded along the dotted segments to make a rectangular prism. Find the volume in cubic inches.

② A room is 18 feet long, 13 feet wide, and 9 feet high. The four walls and ceiling are to be painted. There are two windows, each 3 feet by 4 feet, a window which measures 3½ feet by 6 feet, and three doors which each measure 3 feet by 7 feet. The windows and doors do not get painted. Find the area to be painted, in square feet.

③ It takes 10 seconds to inflate a balloon which is 8 inches in diameter. How many seconds would it take to inflate a balloon which is 24 inches in diameter, if inflated at the same rate? Assume that the balloon is in the shape of a sphere.

ANSWERS

1. _____ cu. in.
2. _____ sq. ft.
3. _____ seconds

Category 3
Mystery
March, 1994

① Find the value of Δ if

$0 \rightarrow 0$
$1 \rightarrow 4$
$2 \rightarrow 16$
$3 \rightarrow 36$
$4 \rightarrow \Delta$

② Ben raises mice and birds. He counted all the heads and got 24. He counted all the feet and got 82. How many mice does Ben have?

③ What number in the sequence $\{11, 20, 29, 38, \ldots\}$ is between 710 and 720?

ANSWERS

1. _____
2. _____
3. _____

Category 4
Arithmetic
March, 1994

① The median of nine consecutive odd integers is 47. What is the sum of the smallest and the largest of those nine integers?

② Two cubical dice are rolled. A cubical die has 6 faces, numbered from 1 through 6. What is the probability that the sum of the numbers on the top faces of the dice is at least 5 (5 or more)? Express your answer as a fraction in lowest terms.

③ Four peanuts and seven cashews were left in the can. Beth selects a nut at random. If she selects a peanut, she will eat it. If she selects a cashew, she will put it back into the can and make another selection, at random. What is the probability that Beth selects a peanut on either her first or second try? Express your answer as a fraction in lowest terms.

ANSWERS

1. _____

2. _____

3. _____

Category 5
Algebra
March, 1994

① The equation $N^2 - 24 = 5N$ has two solutions. What is the value of the smaller of the two solutions?

② The equation $x(x-7) = 18$ has two solutions. What is the sum of those solutions?

③ If an object is thrown or hit into the air with an initial upward velocity (beginning speed) of R inches per second, then its distance, D inches, above its starting point at time T seconds after it was thrown or hit is approximately $D = RT - 5T^2$. As André hits a tennis ball into the air with an initial upward velocity of 27 inches per second, with the ball being hit at a point 18 inches above the ground, after how many seconds will the ball hit the ground?

ANSWERS

1. _____
2. _____
3. _____ sec.

Category 6
Team Questions
March, 1994

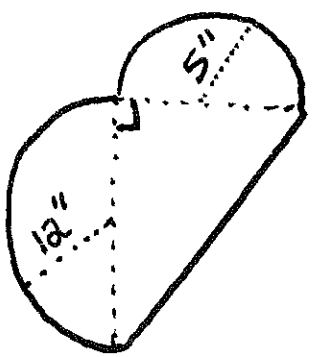
① Find the largest of three consecutive integers whose product is 17,550.

② Solve for W: $\sqrt{39+\sqrt{88+\sqrt{W}}} = 7$

③ The price of an item is reduced by 20 percent. The reduced price must then be increased by C percent to return to the original price. Find the value of C.

④ Kim, Jim, and Tim are business partners. Jim owns $\frac{3}{7}$ of the business. Kim owns twice as much as Tim. What is the numerator of the fraction which represents the part of the business which Kim owns, if that fraction is in lowest terms?

⑤



Find the value of 100Y, if Y represents the perimeter of the figure to the left. Use $\pi \approx 3.14$. The curved parts are semicircles whose radii are 5" and 12" respectively. The perimeter should be given in inches.

⑥ Evaluate: $\dfrac{\left(\frac{E}{\sqrt{B}}\right)\left(\sqrt[3]{AD}\right)}{C}$

ANSWERS

1. _____ = A
2. _____ = B
3. _____ = C
4. _____ = D
5. _____ = E
6. _____

Solution Key – March, 1994

Category 1 NUMBER THEORY

① 178

② 3.3×10^{21}

③ 21112

① $\underline{2}\ \underline{0}\ \underline{1}\ \underline{2}\ \underline{1}$

81 27 9 3 1

$2(81) = 162$
$1(9) = 9$
$2(3) = 6$
$1(1) = 1$
178

② $\dfrac{0.00297 \times 10^{42}}{900 \times 10^{15}} = \dfrac{29.7 \times 10^{38}}{9 \times 10^{17}} = 3.3 \times 10^{21}$

③ First change to a base 10 numeral, then to base 4:

$\underline{7}\ \underline{3}\ \underline{4}$

81 9 1

$7(81) = 567$
$3(9) = 27$
$4(1) = 4$
$598_{\text{Base 10}}$

$598 \div 256 = 2$ R 86
$86 \div 64 = 1$ R 22
$22 \div 16 = 1$ R 6
$6 \div 4 = 1$ R 2

$\therefore 734_{\text{Base 9}} = 598_{\text{Base 10}} = 21112_{\text{Base 4}}$

Category 2 GEOMETRY

① 1560

② 684

③ 270

① $V = lwh$
$= 26(5)(12)$
$= 1560$ cubic inches

② Ceiling: $18 \times 13 = 234$
Walls: $9 \times 13 = 117$
$9 \times 13 = 117$
$9 \times 18 = 162$
$9 \times 18 = 162$
792

Windows: $3 \times 4 = 12$
$3 \times 4 = 12$
$3\frac{1}{2} \times 6 = 21$
Doors: $3 \times 7 = 21$
$3 \times 7 = 21$
$3 \times 7 = 21$
108

$792 - 108 = \boxed{684}$ sq. ft.

③ The larger balloon has a diameter 3 times as long as the smaller, and therefore has a volume 3^3 times as large, or 27 times, and it would take 27 times as much time to inflate it. $27 \times 10 = 270$ seconds.

Category 3 Mystery

① 64

② 17

③ 713

① $0 \rightarrow 0 \quad (0^2)$
$1 \rightarrow 4 \quad (2^2)$
$2 \rightarrow 16 \quad (4^2)$
$3 \rightarrow 36 \quad (6^2)$
$4 \rightarrow 64 \quad (8^2)$

② A pre-algebra student may use a chart to organize a guess + check system: This strategy keeps the # of heads constant at 24.

# of heads			# of feet			
Birds	Mice	Total	Birds	Mice	Total	
20	4	24	40	16	56	Too few feet - increase the # of mice
10	14	24	20	56	76	
8	16	24	16	64	80	
7	(17)	24	14	68	82	Bingo!

③ The numbers in the sequence are two more than multiples of 9. Look for a number between 710 and 720 which is divisible by 9 (which is 711), then add 2. $711 + 2 = 713$.

Category 4 Arithmetic

① 94

② $\frac{5}{6}$

③ $\frac{72}{121}$

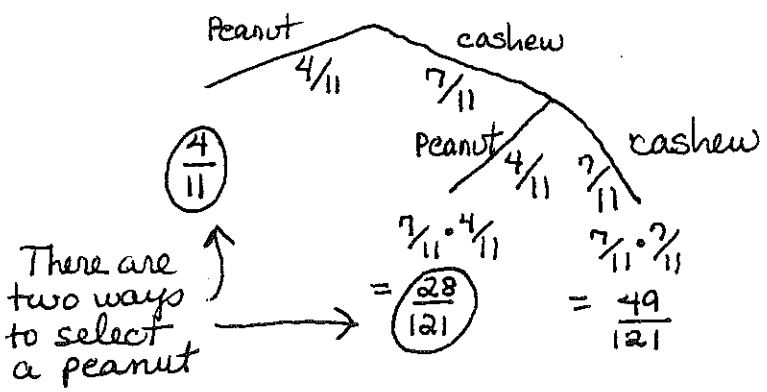
① 39, 41, 43, 45, 47, 49, 51, 53, 55
↑ median

$$\begin{array}{r} 39 \\ +55 \\ \hline (94) \end{array}$$

(which is double the median!)

② There are 36 possible outcomes, and 30 of those have a sum of 5 or more. $\frac{30}{36} = \frac{5}{6}$.

③ A tree diagram may help:



Peanut (1st): $\frac{4}{11} = \frac{44}{121}$

Peanut (2nd): $\frac{28}{121} = \frac{28}{121}$

Sum = $\frac{72}{121}$

Category 5
Algebra

① −3

② 7

③ 6

① $N^2 - 24 = 5N$

$N^2 - 5N - 24 = 0$

$(N-8)(N+3) = 0$

$N = 8$ or $N = -3$

The smaller solution is −3.

② $x(x-7) = 18$

$x^2 - 7x = 18$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$x = 9$ or $x = -2$

The sum of the two solutions is $9 + (-2) = \boxed{7}$

③ $D = -18$, since the ball hits the ground 18 inches below the starting point!

$D = RT - 5T^2$

$-18 = 27T - 5T^2$

$5T^2 - 27T - 18 = 0$

$(5T+3)(T-6) = 0$

$5T + 3 = 0$ or $T - 6 = 0$

$T = -\frac{3}{5}$ or $T = 6$

∴ The ball hits the ground after 6 seconds.

($T = -\frac{3}{5}$ is an extraneous solution, and represents a point on the parabolic curve symmetric to the point where the ball hits the ground, but occurs at a time prior to the ball being hit.)

Category 6
Team Questions

① 27

② 144

③ 25

④ 8

⑤ 7938

⑥ 158.76

① The average of the three integers would be approx. $\sqrt[3]{17{,}550}$. By a little trial and error, the three integers are 25, 26, 27, with 27 being the largest.

② $\sqrt{39 + \sqrt{88 + \sqrt{w}}} = 7$

$39 + \sqrt{88 + \sqrt{w}} = 49$ (square both sides)

$\sqrt{88 + \sqrt{w}} = 10$

$88 + \sqrt{w} = 100$ (square both sides)

$\sqrt{w} = 12$

$w = 144$

Category 6 (continued)

③ Let x = the original price
N = the percent by which the reduced price must be increased to return to the original price

(original price) − (20% discount) = reduced price

$$x - .2x = .8x$$

(Reduced price) + (a percent of reduced price) = original price

$$.8x + N(.8x) = x$$
$$N(.8x) = .2x$$
$$N = \frac{.2x}{.8x}$$
$$N = .25 \text{ or } 25\%$$

If C percent = 25 percent, then $C = 25$.

④ Let T = the fraction of the business which Tim owns
$2T$ = Kim's share of the business

$$T + 2T + \frac{3}{7} = 1$$
$$3T + \frac{3}{7} = 1$$
$$3T = \frac{4}{7}$$
$$T = \frac{4}{21}$$
$$2T = \frac{8}{21}$$

→ Kim's share ($2T$) is $\frac{8}{21}$ of the business.

The numerator is 8.

Category 6 (again continued)

⑤ Use the Pythagorean Theorem to find the diagonal length, where the diameters of the semicircles (10" and 24") are the legs:

$$a^2+b^2=c^2$$
$$10^2+24^2=c^2$$
$$100+576=c^2$$
$$676=c^2$$
$$26=c$$

Perimeter = ⌒ + (+ / = Y

$$= \pi r_1 + \pi r_2 + c$$
$$\approx 3.14(5) + 3.14(12) + 26$$
$$\approx 15.7 + 37.68 + 26$$
$$\approx 79.38''$$

The question asks for the value of 100Y:

$$100(79.38) = \boxed{7938}$$

⑥
$$\frac{\left(\frac{E}{\sqrt{B}}\right)\left(\sqrt[3]{AD}\right)}{C}$$

$$= \frac{\left(\frac{7938}{\sqrt{144}}\right)\left(\sqrt[3]{27\cdot 8}\right)}{25}$$

$$= \frac{\left(\frac{7938}{12}\right)\left(\sqrt[3]{216}\right)}{25}$$

$$= \frac{(661.5)(6)}{25}$$

$$= \frac{3969}{25}$$

$$= 158.76$$