

Category 1
Number Theory
February, 1994

For all categories of
today's contest, you may
use a **CALCULATOR**.

(1 point)

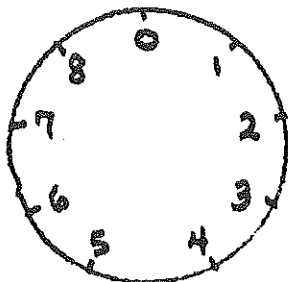
① Find the value of the 268th term of this sequence:

1st term 2nd term 3rd term 4th term

9 21 33 45 ...

(2 points)

② Mickey lives on the planet Pluto, and has invented a modulo-9 clock to celebrate the fact that Pluto is the ninth planet of the solar system. He left Pluto at 3 o'clock (mod 9), travelled to earth, then rushed back to Pluto to report of his visit to Disney World! The round trip lasted 42,575 hours. At what time, mod 9, did Mickey return to Pluto? Express your answer as a whole number.



(3 points)

③ Find the sum of the first 94 terms in this sequence:

19, 26, 33, 40, 47, ...

ANSWERS

1. _____

2. _____

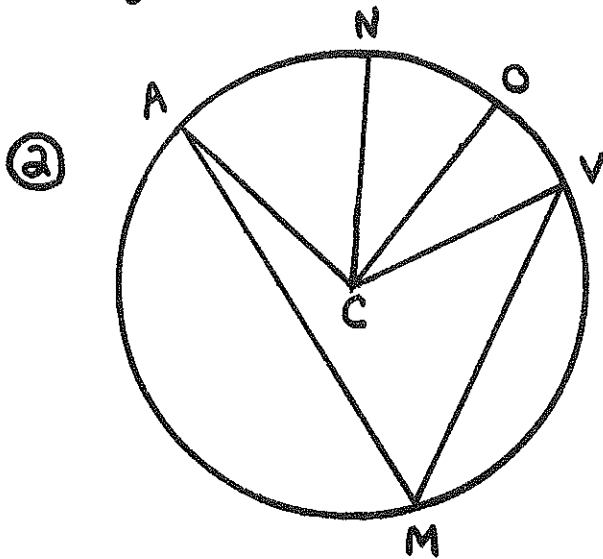
3. _____

Category 2
 Geometry
 February, 1994

You may use a CALCULATOR.

(1 point)

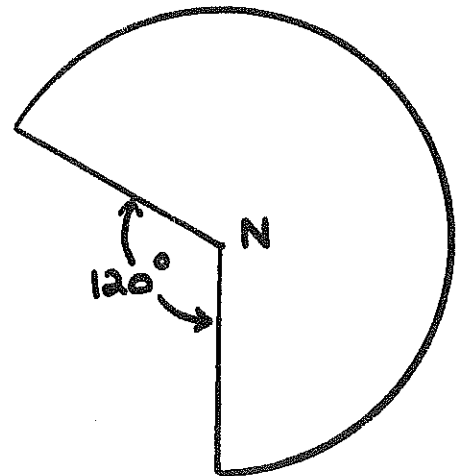
- ① The area of a circular dish is 88.2026 square inches. How many inches are in the diameter of the dish? Use $\pi \approx 3.14$.



Angle ACN measures 53° , angle OCV measures 38° , and angle AMV measures 67° . How many degrees are in the measure of angle NCO? (2 points)

(3 points)

- ③ The area of the figure at the right is 36.94992 square meters, and is a portion of a circle whose center is at N. What is the perimeter of the figure? Use $\pi \approx 3.142$.



ANSWERS

1. _____ inches
 2. _____ degrees
 3. _____ meters

Category 3
Mystery
February, 1994

You may use a CALCULATOR.

(1 point)

- ① Carol has 60 boxes of candy. If she sells 15% of the boxes, how many boxes will be left over?

(2 points)

- ② Jim has a collection of quarters and dimes, a total of 30 coins in all. The total value of the coins is more than \$5.20. What is the least number of quarters that Jim has? (At least how many of the coins are quarters?)

(3 points)

- ③ Shawna shovelled one-third of the driveway before lunch. After lunch, her brother, Shawn, shovelled $\frac{2}{5}$ of the rest of the driveway, saving the remaining portion for his father, Don. What fraction of the driveway was saved for Don to shovel? Express your answer as a fraction in lowest terms.

ANSWERS

1. _____

2. _____

3. _____

Category 4
Arithmetic
February, 1994

You may use a CALCULATOR.

(1 point)

- ① Including a 5% sales tax, how much would it cost to buy an aquarium marked \$59.95? Round to the nearest hundredth of a dollar.

(2 points)

- ② Ben earns a fixed salary plus a commission based on how many dollars worth of shoes he sells. He earns a 6% commission this month for selling \$3140 worth of shoes. His total paycheck this month is \$521.17. What is his fixed salary?

(3 points)

- ③ A \$100 savings bond may be purchased for half its face value. If the bond earns ~~an~~ 8% interest compounded yearly, in how many years will the bond be worth \$100? (whole number of years)

ANSWERS

1. \$
2. \$
3. years

Category 5
Algebra
February, 1994

You may use a CALCULATOR.

(1 point)

- ① The sum of four consecutive even integers is 268. What is the sum of the smallest and largest of them?

(2 points)

- ② Harold is three times as old as Maude. If Harold were five years older, he would be 38 years older than Maude was three years ago. How old is Harold now?

(3 points)

- ③ A cargo ship must average 45 Km per hour to make its 14-hour run on schedule. During the first four hours, bad weather forced the captain to reduce speed to 30 Km per hour. What should the average speed of the ship be for the rest of the trip in order to arrive on schedule?

ANSWERS

1. _____

2. _____ years

3. _____ Km/hr.

Category 6
Team Questions
February, 1994

You may use a CALCULATOR.
Each problem is worth 6 points

- ① Calculate: $(713 \bmod 5) + (2468 \bmod 9)$
- ② The area of a circle is 9856 ft.^2 What is its circumference? Use $\pi \approx \frac{22}{7}$.
- ③ RR Hood went to her grandmother's house, averaging 30 miles per hour. She returned home averaging 50 miles per hour. What was her average rate of speed for the round trip?
- ④ How many ounces of water must be added to 80 ounces of a 35% acid solution to dilute it to become a 10% solution?
- ⑤ What is the denominator of the fraction which is the next term in this geometric sequence:
 $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \underline{\hspace{1cm}}$ (the fraction should be in lowest terms)
- ⑥ Use the answers of question # 1-5 to evaluate the following expression:

<u>ANSWERS</u>	
1. _____	= A
2. _____	= B
3. _____	= C
4. _____	= D
5. _____	= E
6. _____	

$$\sqrt{\frac{B - 2C - 34}{E} + \frac{D}{A}}$$

Solution Key - February, 1994

Jack

Category 1

- ① 3213 ① Each term is 3 less than a multiple of 12.
 $268(12) - 3 = 3216 - 3 = 3213$
- ② 8 ② $42,575 \div 9 = 4730$, remainder 5.
 $3 + 5 = 8$
- ③ 32,383 ③ The 94th term is $94(7) + 12 = 670$
 The sum is $47(19 + 670) = 47(689)$
 $= 32,383$

Category 2

- ① 10.6 ① $A = \pi r^2 \Rightarrow 88.2026 \approx 3.14 r^2 \Rightarrow r^2 = 28.09$
 $\Rightarrow r \approx 5.3 \Rightarrow \text{diameter} \approx 10.6$
- ② 43 ② If the inscribed angle $AMV = 67^\circ$, then the
 central angle $ACV = 2(67)$ or 134°
 $53 + 38 + x = 134$, so $x = 43^\circ$
- ③ 25.9952 ③ Be careful - here, $\pi \approx 3.142$!
 The figure is $\frac{2}{3}$ of a circle, so area $= \frac{2}{3}\pi r^2$
 $36.94992 \approx 3.142(r^2)(\frac{2}{3}) \Rightarrow r^2 \approx 11.76(\frac{3}{2})$
 $\Rightarrow r^2 \approx 17.64 \Rightarrow r \approx 4.2 \Rightarrow d = 8.4$
 Perimeter $= \frac{2}{3}$ of circumference + 2 · radius
 $\approx \frac{2}{3}(3.142)(8.4) + 2(4.2)$
 $\approx 17.5952 + 8.4$
 ≈ 25.9952

Category 3

- ① 51 ① $85\% \text{ of } 60 = \frac{85}{100} \cdot 60 = 51$
- ② 15 ② Try a chart. Keep total
 # of coins constant at 30:
- | # quarters | # dimes | Value |
|------------|---------|--------|
| 10 | 20 | \$4.50 |
| 12 | 18 | 4.80 |
| 14 | 16 | 5.10 |
| 15 | 15 | 5.25 |
- ③

Shawna	Shawn $\frac{2}{5}$ of $\frac{2}{3} = \frac{4}{15}$
$\frac{1}{3}$	Don $1 - (\frac{1}{3} + \frac{4}{15})$ $= 1 - \frac{9}{15} = \frac{6}{15} = \frac{2}{5}$

Solution Key - continued

Category 4

- ① 62.95
- ② 332.77
- ③ 10

① Tax = $.05(59.95) = 2.9975 \approx 3.00$
 Cost = $59.95 + 3.00 = 62.95$

② Commission = $.06(3140) = 188.40$
 Salary = $521.17 - 188.40 = 332.77$

③

End of year #	Value
0	50.00
1	54.00
2	58.32
3	62.9856
4	68.024448
5	73.466403
6	79.343715
7	85.691212
8	92.546508
9	99.950228
10	107.94624

Category 5

- ① 134
- ② 45
- ③ 51

① $x, x+2, x+4, x+6$ are four consecutive integers.

$$x + x+2 + x+4 + x+6 = 268$$

$$4x + 12 = 268$$

$$4x = 256$$

$$x = 64$$

sum of smallest + largest is $64 + 70 = 134$

② Let $M =$ Maude's age
 $3M =$ Harold's age

$$3M + 5 = (M - 3) + 38$$

$$3M + 5 = M - 3 + 38$$

$$M = 15 \quad \therefore 3M = 45$$

③ The run is $14(45)$ or 630 Km.

First four hours: $4(30) = 120$ Km.

There are 510 Km remaining, $510 \div 10 = 51$
 and 10 hours.

Category 6

- ① 5
- ② 352
- ③ 37.5
- ④ 200
- ⑤ 27
- ⑥ 7

① $3+2 = 5$

② $A = \pi r^2$
 $9856 = \frac{22}{7} r^2$
 $3136 = r^2$
 $56 = r$

$$C = 2\pi r$$

$$= 2\left(\frac{22}{7}\right)(56)$$

$$= 352$$

③ $30T = D$
 $T = D/30$ (to)

$$50T = D$$

$$T = D/50$$
 (from)

Average rate = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{2D}{\frac{D}{30} + \frac{D}{50}}$$

$$= 37.5$$

Solution Key - continued

③ (continued)

A simpler technique would be to let the distance be some multiple of 30 and 50, say 150. It would take 5 hours to get to grandma's house, and 3 hours to return.

$$\frac{\text{Total distance}}{\text{Total time}} = \frac{300}{8} = 37.5$$

④ # of ounces of acid in the solution = $.35(80)$
= 28

28 is 10% of 280, the total # of ounces required.

$$280 - 80 = 200$$

⑤ To get from one term to the next, multiply by $\frac{2}{3}$. Know that this is a geometric sequence. To find the # multiplied, divide any term by the preceding term. Example: $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$.

$$\text{Now } \frac{2}{9} \times \frac{2}{3} = \frac{4}{27}$$

$$\textcircled{6} \quad \sqrt{\frac{352 - 2(37.5) - 34}{27} + \frac{200}{5}}$$

$$= \sqrt{9 + 40}$$

$$= \sqrt{49}$$

$$= 7$$