Solution Key – January, 1994

Meet 3
Jan 1994

CATEGORY 1

① −1

$20 < C < 30$: $21+22+24+25+26+27+28 = 173$

$80 < P < 90$: $83+89 = 172$

(sum of primes) − (sum of composites) = $172 - 173 = -1$

② 180

Find the LCM of 12, 18, 20.

$12 = 2^2 \cdot 3$
$18 = 2 \cdot 3^2$
$20 = 2^2 \cdot 5$

$LCM = 2^2 \cdot 3^2 \cdot 5$
$= 4 \cdot 9 \cdot 5$
$= 180$

③ 81

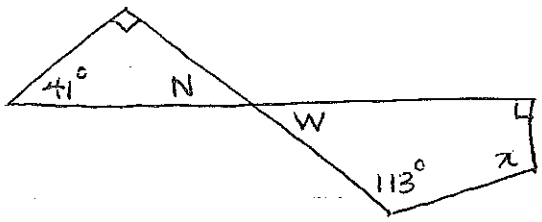
Remember that $GCF(A,B) \cdot LCM(A,B) = A \cdot B$

$$\frac{A \cdot B}{24} = \frac{36 \cdot 54}{24} = \frac{(2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3 \cdot 3)}{2 \cdot 2 \cdot 2 \cdot 3}$$

$= 81$

CATEGORY 2

① 108 (degrees)



Sum of angles of a triangle is 180°.
∴ $N = 180 - (90+41)$
$= 180 - 131$
$= 49°$

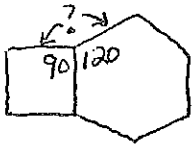
Vertical (opposite) angles are congruent, so $W = 49°$

The sum of the angles of a quadrilateral is 360°, so

$x = 360 - (113+49+90)$
$= 360 - (252)$
$= 108°$

② 150 (degrees)

Each angle of the square measures 90°, while each measure of the hexagon measures $\frac{180(6-2)}{6} = \frac{180(4)}{6} = 120°$



$? = 360 - (90+120)$
$= 360 - 210$
$= 150°$

③ 144 (cm^2)

The height and base of $\triangle MAH$ are known, so area = $\frac{8(6)}{2} = 24$

Neither height nor base of $\triangle ATH$ are known, but both can be found by using the Pythagorean Theorem.

$6^2 + 8^2 = (HT)^2$
$36 + 64 = (HT)^2$
$100 = (HT)^2$

$\therefore HT = 10$

$10^2 + (AT)^2 = 26^2$
$100 + (AT)^2 = 676$
$(AT)^2 = 576$
$\therefore AT = 24$

area of $\triangle ATH = \frac{10(24)}{2} = 120$

Total area = $24 + 120$
$= 144$

Solution Key - January, 1994 (continued)

CATEGORY 3

① (#) 222

Selim's per hour rate of pay is $\frac{\#120}{40\text{ h}} = \#3/\text{h}.$
Angela's rate is twice Selim's rate $= 2 \times 3 = \#6/\text{h}.$
Angela earns $\#6/\text{h}. \times 37\text{ h}. = \#222.$

② 19

Students with little or no knowledge of algebra may use a chart to organize data and adjust their guesses. Keep the total # of creatures constant at 72.

# of 3-legged creatures	# of legs of 3-legged creatures	# of 5 legged creatures	# of legs of 5-legged creatures	Total # of legs
40	120	32	160	280
30	90	42	210	300

We need more 3-legged creatures to reduce the total # of legs.

45	135	27	135	270
50	150	22	110	260
53	159	(19)	95	254

A student with some algebra experience may use equations:

Let x = # of 3-legged creatures
$72 - x$ = # of 5-legged creatures

$$3x + 5(72 - x) = 254$$
$$3x + 360 - 5x = 254$$
$$-2x = -106$$
$$x = 53$$
$$72 - x = 19$$

③ 80

	# of candy bars before trade	# after trade
Lucy	$2x$	$2x - 24$
Charlie	x	$x + 24$

$$2x - 24 + 8 = x + 24$$
$$2x - 16 = x + 24$$
$$x = 40$$
$$2x = 80$$

~~$x + 24 + 8 = 2x - 24$~~
~~$x + 32 = 2x - 24$~~
~~$56 = x$~~
~~$112 = 2x$~~

OR organize a chart like for problem # 2.

Solution Key - January, 1994 (continued)

CATEGORY 4

① $\frac{364}{9}$ only

$3^3 + 3^2 + 3^1 + 3^0 + 3^{-1} + 3^{-2}$
$= 27 + 9 + 3 + 1 + \frac{1}{3} + \frac{1}{9}$
$= 40 + \frac{3}{9} + \frac{1}{9}$
$= 40\frac{4}{9}$
$= \frac{364}{9}$ (The problem requires an improper fraction.)

② 2

$= \frac{5+10-2+3}{2 \cdot 4} = \frac{16}{8} = 2$

③ 125

$\sqrt[4]{25^2} = \sqrt[4]{625} = 5$; $5^{-2} = \frac{1}{25}$; $\sqrt{\frac{1}{25}} = \frac{1}{5}$;

$\frac{1}{5}^{-1} = 5$; $5^3 = 125$

CATEGORY 5

① 44

$|6| + 3|-4| - 7|-2| + 8|5| - |0|$
$= 6 + 3(4) - 7(2) + 40 - 0$
$= 6 + 12 - 14 + 40 - 0$
$= 44$

② $\{-4, -3, -2, -1\}$

$-7 - 3(2x - 5) < 38$
$-7 - 6x + 15 < 38$
$-6x + 8 < 38$
$-6x < 30$
$x > -5$

If the domain is {neg. integers} the solution set is $\{-4, -3, -2, -1\}$

③ 12 (feet)

$\frac{\text{height of object}}{\text{length of shadow}}$ $\quad \frac{5\frac{2}{3}}{4} = \frac{17}{x}$

$5\frac{2}{3}x = 4(17)$
$\frac{17}{3}x = 68$
$\frac{3}{17} \cdot \frac{17}{3}x = \frac{3}{17} \cdot 68$
$x = 12$ feet

CATEGORY 6
Team Questions

① 464

$40 < P < 60$: $41+43+47+53+59 = 243$

$40 < C < 60$: $42+44+45+46+48+49+50+51+52+54+55+56+57+58 = 707$

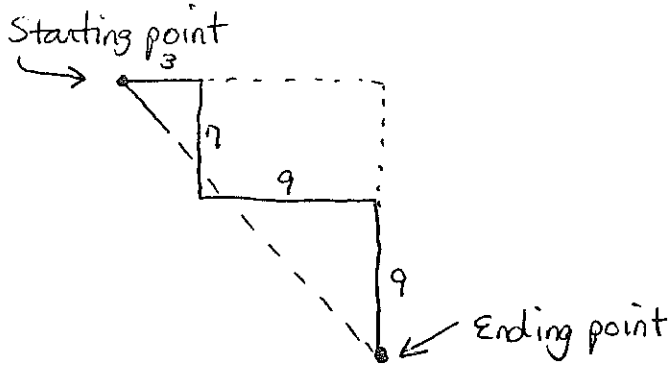
(sum of composites) – (sum of primes) $= 707 - 243 = 464$

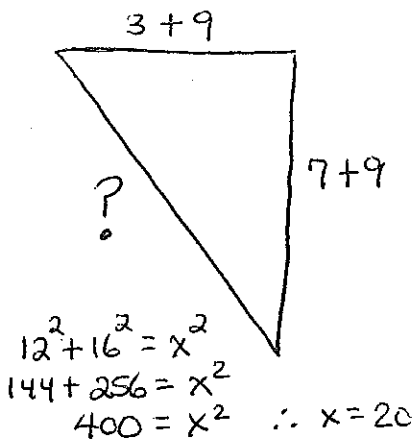
② 24

one angle (interior) $= \frac{180(15-2)}{15} = 12(13) = 156°$

one exterior angle $= 180 - 156 = 24°$

③ 20 (miles)





$12^2 + 16^2 = x^2$

$144 + 256 = x^2$

$400 = x^2 \quad \therefore x = 20$

④ 88

$\frac{7\frac{3}{4}}{11} = \frac{62}{Y}$

$7\frac{3}{4}Y = 11 \cdot 62$

$7\frac{3}{4}Y = 682$

$\frac{31}{4}Y = 682$

$\frac{4}{31} \cdot \frac{31}{4}Y = \frac{4}{31} \cdot 682$

$Y = 88$

⑤ 5

$5^0 + 4^1 + 3^2 - 2^3 - 1^4 - 0^5 = 1 + 4 + 9 - 8 - 1 - 0 = 5$

⑥ 2

$$\frac{D - \frac{3A}{B} + C}{\left(\frac{1}{E}\right)^{-2}}$$

$$= \frac{88 - \frac{3(464)}{24} + 20}{\left(\frac{1}{5}\right)^{-2}}$$

$$= \frac{88 - 58 + 20}{25}$$

$$= \frac{50}{25}$$

$$= 2$$

Category 1
Number Theory
January, 1994

For each category today, place your answers into the answer spaces in the lower left corner of the paper.

(worth 1 point)

① Subtract the sum of all composite numbers which are greater than 20 and less than 30 from the sum of all prime numbers between 80 and 90.

(worth 2 points)

② Tim eats at MacDonald's every 12 days. Felicia eats there every 18 days, while Aaron eats there every 20 days. All three people are eating there today. In how many more days will they all be eating at MacDonald's (when is the next time they will eat there)?

(worth 3 points)

③ A = the GCF (greatest common factor) of 36 and 54.
B = the LCM (lowest common multiple) of 36 and 54.
Find the value of $\frac{A \cdot B}{24}$.

ANSWERS

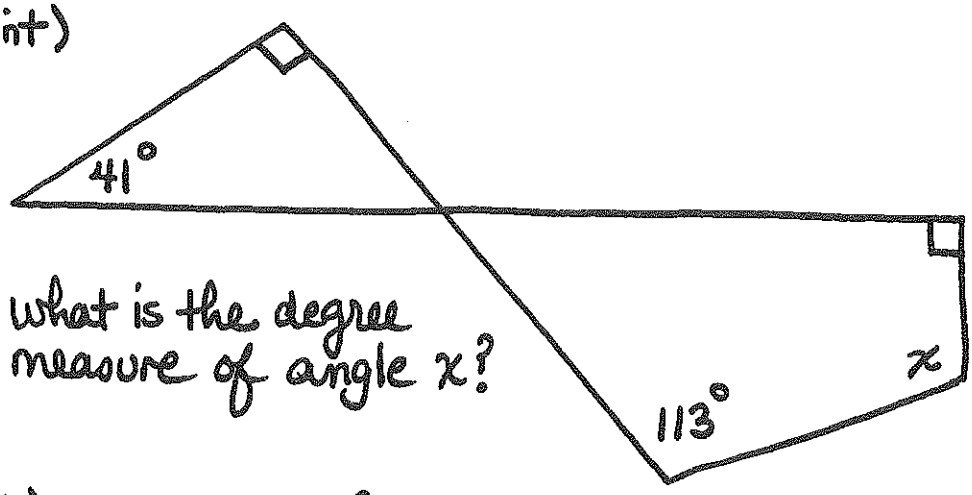
① _____

② _____

③ _____

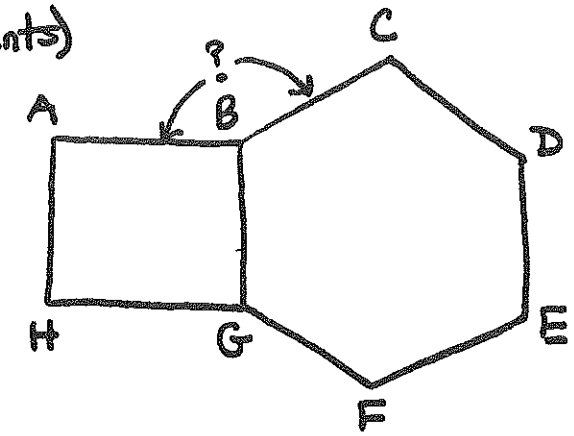
Category 2
Geometry
January, 1994

(1 point)

① 

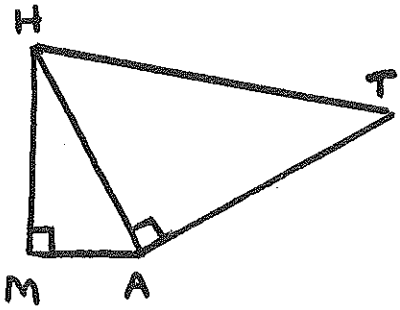
what is the degree measure of angle x?

(2 points)

② 

ABGH is a square.
BCDEFG is a regular hexagon.
What is the measure of angle ABC?

(3 pts)

③ 

HM = 8 cm
AM = 6 cm
HT = 26 cm

what is the area, in square centimeters of polygon MATH?

ANSWERS

① _____ degrees
② _____ degrees
③ _____ sq. cm

Category 3
Mystery
January, 1994

(1 point)

① Selim worked for 40 hours and earned $120. Angela earned twice as much per hour. How much money did Angela earn for working 37 hours?

(2 points)

② Katie took a walk on the planet Zortron and saw 72 creatures. The three-legged creatures were blue and the five-legged creatures were red. All 72 creatures were either blue or red. How many red creatures were there, if Katie counted a total of 254 legs?

(3 points)

③ Lucy has twice as many candy bars as Charlie. If she gave 24 candy bars to Charlie, then Charlie would have eight more than Lucy. How many candy bars did Lucy have originally?

ANSWERS

① _____

② _____

③ _____

Category 4
Arithmetic
January, 1994

(1 point)

① Evaluate: $3^3+3^2+3^1+3^0+3^{-1}+3^{-2}$ and express your answer as an improper fraction in lowest terms.

(2 points)

② Evaluate: $\dfrac{\sqrt{25}+\sqrt[3]{1000}-\sqrt[5]{32}+\sqrt[4]{81}}{(\sqrt[6]{64})(\sqrt[3]{64})}$

(3 points)

③ Evaluate: $\left(\left(\sqrt{\left(\sqrt[4]{25^2}\right)^{-2}}\right)^{-1}\right)^3$

ANSWERS

① _____

② _____

③ _____

Category 5
Algebra
January, 1994

(1 point)

① Evaluate: $|6| + 3|-4| - 7|-2| + 8|5| - |0|$

② Solve $-7 - 3(2x - 5) < 38$ if the replacement set of x is {negative integers}.

③ If a woman $5\frac{2}{3}$ feet tall casts a shadow which is 4 feet long, then how many feet long is a shadow cast by a telephone pole which is 17 feet high?

ANSWERS

① _____

② _____

③ _____

Team Questions
January, 1994

(6 points each)

① Subtract the sum of all prime numbers between 40 and 60 from the sum of all composite numbers between 40 and 60 (not including 40 or 60).

② Find the measure of one exterior angle of a regular pentadecagon (15-sided polygon).

③ Jon walked 3 miles East, then 7 miles South, then 9 miles East, then 9 miles South. How far is he from his starting point, in miles?

④ Solve for Y: $\dfrac{7\frac{3}{4}}{11} = \dfrac{62}{Y}$

⑤ Evaluate: $5^0 + 4^1 + 3^2 - 2^3 - 1^4 - 0^5$

⑥ Use the answers from questions #1-5 to evaluate the following:

$$\frac{D - \frac{3A}{B} + C}{\left(\frac{1}{E}\right)^{-2}}$$

ANSWERS

① _____ = A
② _____ = B
③ _____ = C
④ _____ = D
⑤ _____ = E
⑥ _____