CATEGORY 1
NUMBER THEORY
MARCH 1, 1990

Meet 4

1. _____

2. _____ (three)

3. _____ (five)

1. FIND THE THREE DIGIT COUNTING NUMBER WHOSE DIGITS ARE ALL EVEN AND ALL DIFFERENT. THE NUMBER IS DIVISIBLE BY FOUR. THE SUM OF THE DIGITS IS TWELVE. THE SUM OF THE UNITS AND TENS DIGITS IS EQUAL TO THE HUNDREDS DIGIT.

2. $1,100,111_{(two)} =$ _____ (three)

3. FIND THE QUOTIENT IN BASE FIVE.

$$32_{(five)} \overline{)\,3031_{(five)}}$$

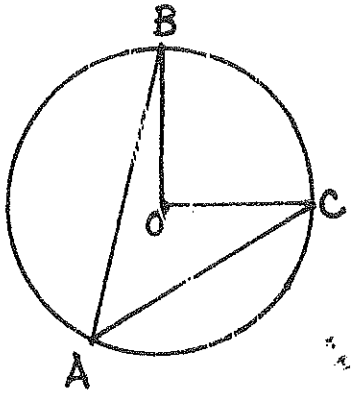
CATEGORY 2
GEOMETRY
MARCH 1, 1990

1. _____ °

2. _____ $in^2$

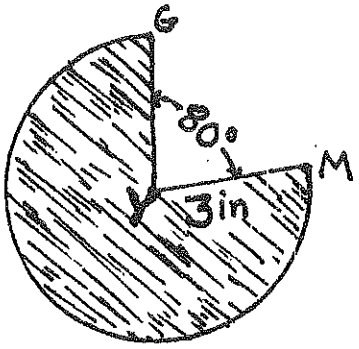
3. _____ $in^2$

1.



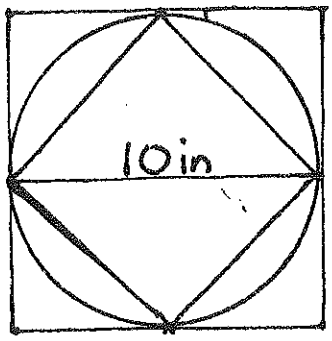

IF POINT O IS THE CENTER OF THE CIRCLE AND $m\angle BOC = 90°$ WHAT IS THE MEASURE OF $\angle BAC$?

2.




USING 3.14 FOR THE VALUE OF $\pi$ FIND THE AREA OF THE SHADED SECTOR.
$m\angle GYM = 80°$
$r = 3in$

3.




IF THE DIAMETER OF THE CIRCLE IS 10IN, WHAT IS THE DIFFERENCE IN THE AREA OF THE CIRCUMSCRIBED SQUARE AND THE INSCRIBED SQUARE?

CATEGORY 3
MYSTERY
MARCH 1, 1990

1. _____

2. _____

3. _____

1. WHAT NUMBER BETWEEN 100 AND 150 HAS EXACTLY THREE FACTORS?

2. SIMPLIFY (GIVE YOUR ANSWER IN FRACTION FORM; $\frac{a}{b}$)

$$\left(1-\tfrac{1}{3}\right)\left(1-\tfrac{1}{4}\right)\left(1-\tfrac{1}{5}\right)\left(1-\tfrac{1}{6}\right)\cdots\left(1-\tfrac{1}{100}\right)$$

3. A TEST CONTAINS 42 QUESTIONS SOME OF WHICH ARE WORTH 2 POINTS AND THE REST OF WHICH ARE WORTH 3 POINTS. IF A PERFECT SCORE IS 100, HOW MANY 3 POINT QUESTIONS ARE ON THE TEST?

CATEGORY 4
ARITHMETIC
MARCH 1, 1990

1. $_____

2. $_____

3. $_____

THE USE OF CALCULATORS IS PERMITTED ON THIS TEST.

1. ERNIE EARNS A COMMISSION OF .035% AS A WIDGET SALESMAN. IN 1989 HE SOLD $32.86 MILLION WORTH OF WIDGETS. WHAT WAS HIS COMMISSION?

2. LECRETIA INVESTED $1250 AT 9.6% ANNUAL INTEREST COMPOUNDED MONTHLY. HOW MUCH WAS HER INVESTMENT WORTH AFTER THREE MONTHS? (ROUND YOUR ANSWER TO THE NEAREST PENNY.)

3. MILLIE BOUGHT A DRESS AT THE LIMITED WHICH ORIGINALLY SOLD FOR $108 BUT HAD BEEN DISCOUNTED 33%. CONNIE BOUGHT THE SAME $108 DRESS AT CUMMINGS WHERE IT HAD BEEN SUCCESSIVELY DISCOUNTED $12\frac{1}{2}\%$ AND THEN 20%. HOW MUCH MORE DID CONNIE PAY FOR THE DRESS?

CATEGORY 5
ALGEBRA
MARCH 1, 1990

1. ____, ____, ____
2. ____
3. ____

1. FIND 3 CONSECUTIVE INTEGERS SUCH THAT 3 TIMES THE SMALLEST IS 16 MORE THAN THE LARGEST.

2. A MAN IS 40 YEARS OLD AND HIS SON IS 8 YEARS OLD. IN HOW MANY YEARS WILL THE MAN BE 3 TIMES AS OLD AS HIS SON WILL BE THEN?

3. MATT PAID HIS BILL AT MACDONALD'S WITH ONLY QUARTERS, DIMES AND PENNIES. HE HAD 5 LESS QUARTERS THAN DIMES AND 4 MORE PENNIES THAN QUARTERS. IF HIS BILL WAS $7.38, HOW MANY QUARTERS DID HE USE?

CATEGORY 6
TEAM QUESTIONS
MARCH 1, 1990

1. A= _____
2. B= _____
3. C= _____
4. D= _____
5. E= _____
6. F= _____

1. ALTHOUGH THE AREA OF A CIRCLE IS MEASURED IN SQUARE UNITS AND THE CIRCUMFERENCE IS MEASURED IN UNITS, BOTH THE AREA AND THE CIRCUMFERENCE OF THIS CIRCLE HAVE THE SAME NUMERICAL VALUE. WHAT IS THE RADIUS OF THIS CIRCLE.

2. TELEPHONE AREA CODES ARE MADE UP OF 3 DIGITS.
THE FIRST DIGIT MAY BE 2 THROUGH 9.
THE SECOND IS EITHER 0 OR 1.
THE THIRD CAN BE ANY DIGIT EXCEPT ZERO.
HOW MANY DIFFERENT AREA CODES ARE POSSIBLE?

3. HOW MANY OF THE FOUR DIGIT NUMBERS WHICH USE THE DIGITS 1, 2, 3, 4 ARE DIVISIBLE BY ELEVEN?

4. $12^4 \cdot 15^6$ ENDS IN HOW MANY ZEROS ?

5. SEVERAL PENNIES ARE SORTED INTO PILES ACCORDING TO MINT DATES. THERE ARE 486 IN THE 1989 PILE. THERE ARE 1/2 AS MANY IN THE 1988 PILE AND 2/3 OF THIS NUMBER IN 1987 AND 1/2 OF THIS NUMBER IN 1986 AND 2/3 OF THIS NUMBER IN 1985. IF THIS PATTERN CONTINUES, WHAT IS THE DATE ON THE OLDEST COIN?

6. $$\frac{E-(A+C)}{CA\sqrt{\frac{B}{AC}}} = F$$

# ANSWERS

CAT. 1
NUMBER THEORY

1. 624
2. $10211_{(three)}$
3. $43_{(five)}$

CAT. 2
GEOMETRY

1. $45^\circ$
2. $21.98 in^2$
3. $50 in^2$

CAT 3
MYSTERY

1. 121
2. 1/50
3. 16

CAT 4
ARITHMETIC

1. $11,501
2. $1280.24
3. $3.24

CAT. 5
ALGEBRA

1. 9, 10, 11
2. 8
3. 19

CAT. 6
TEAM

1. A = 2
2. B = 144
3. C = 8
4. D = 6
5. E = 1978
6. F = 41

# SELECTED SOLUTIONS:

## NUMBER THEORY:

2. $1,100,111_2 = 103_{10} = 10211_3$

3 $3031_5 \div 32_5 = 391_{10} \div 17_{10} = 23_{10} = 43_5$

## GEOMETRY:

2. SECTOR IS $\frac{280^\circ}{360^\circ}$ or $\frac{7}{9}$ OF CIRCLE

$\therefore A = \pi r^2 \left(\frac{7}{9}\right)$

$A = 3.14(9)\left(\frac{7}{9}\right)$

$A = 21.98$

3. PYTHAG. TH. GIVES SIDE OF INSCRIBED SQUARE $\sqrt{50}$ OR $5\sqrt{2}$ $\therefore A = 50$
CIRCUMSCRIBED SQUARE $A = 10^2$ OR 100
DIFFERENCE $100 - 50 = 50$

## MYSTERY

1. THE SQUARE OF A PRIME $\therefore 11^2 = 121$

2. $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdots \frac{99}{100} = \frac{2}{100} = \frac{1}{50}$

3. WHILE ALGEBRAIC SOLUTION IS POSSIBLE AN ORGANIZED LISTING, GUESS + TEST, LOOKING FOR PATTERNS ARE VIABLE ALTERNATIVE STRATEGIES RESULTING IN $3 \cdot 18 = 54$ } 100 POINTS
$2 \cdot 23 \quad 46$ }

## ARITHMETIC

1. $.00035 \times 32.86 \times 1{,}000{,}000 = \$11{,}501$

2. $\$1250 \times .096 \div 12(\text{mos.}) = \$10$ int. for 1st mo.
$\$1260 \times .096 \div 12 = \$10.08$ int for 2nd mo.
$\$1270.08 \times .096 \div 12 = 10.16$ in for 3rd mo.
$\$1270.08 + 10.16 = \$1280.24$

3. MILLIE
$\$108 \times .33 = 35.64$
$108 - 35.64 = \boxed{\$72.36}$

CONNIE
$\$108 \times .125 = \$13.50$
$108 - 13.50 = 94.50$
$\$94.50 \times .2 = 18.90$
$\$94.50 - 18.90 = \boxed{\$75.60}$

$\$75.60 - 72.36 = \boxed{\$3.24}$

## ALGEBRA

1. $3x = (x+2) + 16$
2. $40 + x = 3(8+x)$
3. $25(x-5) + 10x + (x-5) + 4 = 738$

## TEAM

2. $8 \cdot 2 \cdot 9 = 144$
3. THOSE STUDENTS WHO UNDERSTAND DIVISIBILITY BY 11 WILL RECOGNIZE THAT ONLY THE FOLLOWING ARE DIVISIBLE

| | | | |
|---|---|---|---|
| 1243 | 4213 | 2134 | 3124 |
| 1342 | 4312 | 2431 | 3421 |

BECAUSE THE SUMS OF THE 1ST + 3RD AND THE 2ND + 4th DIGITS ARE EQUAL -

WITHOUT THIS INFORMATION SOME INTERESTING TEAM STRATEGIES MAY DEVELOP

4. $(2 \cdot 2 \cdot 3)^4 \cdot (5 \cdot 3)^6 = \underbrace{(5 \cdot 2)^6}_{\text{6 zeros}} \cdot 2^2 \cdot 3^4$

5.
1989 - 486
1988 - 243
1987 - 162
1986 - 81
1985 - 54
1984 - 27
1983 - 18
1982 - 9
1981 - 6
1980 - 3
1979 - 2
1978 - 1

6. $\dfrac{1978 - (2+8)}{8 \cdot 2\sqrt{\frac{144}{2 \cdot 8}}} = \dfrac{1968}{48} = \boxed{41}$